

Module - 1

DC Circuits.

1. Brief about ohm's law :-

ohm's law :-

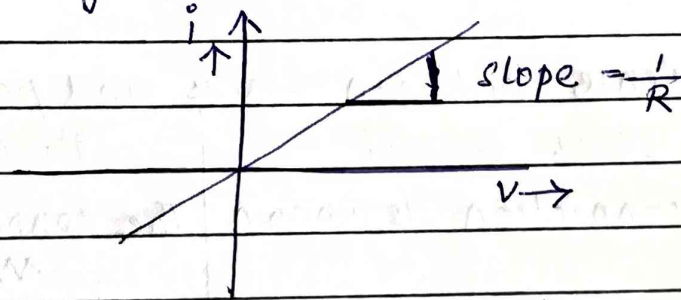
Potential

It states that the difference between the two ends of the conductor is directly proportional to the current flowing through it, provided its temperature and other physical parameters remains unchanged.

$$V \propto I$$

$$V = IR$$

ohm's law in graphical form :-



- * The Voltage is shown as a independent Variable
- * current as dependent Variable
- * Slope of the line is the reciprocal of resistance ($1/R$)
- * $1/R$ is called the conductance
- * The conductor showing straight line $V-i$ characteristic is said to have linear resistance.

Microscopic form of ohm's law :-

By definition, current density is current per unit area.

$$J \equiv \frac{I}{A} = \frac{(V/R)}{A} = \frac{V}{RA} = \frac{V}{(PL/A)A} = \frac{(V/L)}{P} = \frac{E}{P}$$

where $E = V/L$ is the electric field at a point in the conductor. The above equation can be written as

$$E = PJ$$

This is often referred as microscopic form of ohm's law

2. Brief about resistance :-

The flow of current in a conductor is analogous to the flow of water in a pipe.

The electrical resistance of a conductor is directly proportional to its length (L) and inversely proportional to its area of cross section (A)

$$R \propto \frac{L}{A} \quad \text{or} \quad R = \frac{\rho L}{A}$$

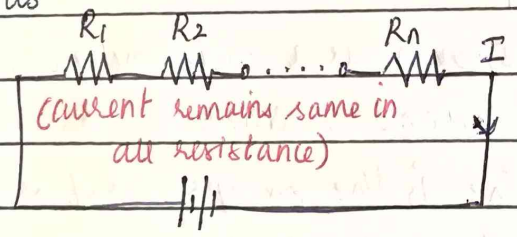
ρ = constant of proportionality or resistivity of the material. unit of resistivity is ohm meter (Ωm)

3. Difference between series and parallel network.

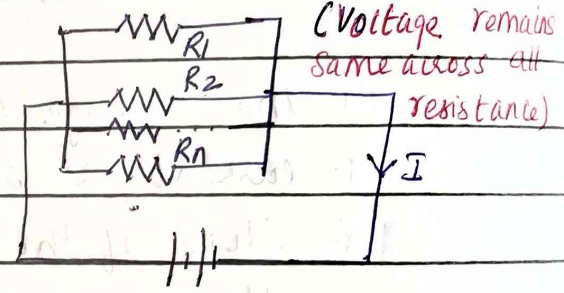
Series circuit

Parallel circuit

1. The connection is shown as



The connection is as shown



2. The same current flows through each resistance

The same voltage exist across all resistance in parallel.

3. The voltage across each resistance is different

The current through each resistance is different

4. The equivalent resistance is $R_{eq} = R_1 + R_2 + \dots + R_n$

The equivalent resistance is $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n}$

5. The equivalent resistance is the largest than each of the resistances in series.

The equivalent resistance is the smaller than the smallest of all resistance in parallel.

4. What are the factors affecting resistance?

1. Length:-

The resistance of a material is directly proportional to the length. Length is denoted by 'l'

2. Cross-sectional area:-

The resistance of a material is inversely proportional to the cross-sectional area of the material. The cross sectional area is denoted by 'a'.

3. The type and nature of the material:-

If the material is a conductor, its resistance is less while if it is insulator, its resistance is very high.

4. Temperature :-

As temperature changes, the value of the resistance of the material changes. So for a certain material at a certain constant temperature we can write a mathematical expression-

$$R \propto \frac{l}{A}$$

The effect of nature of material is considered through the constant of proportionality denoted by 'ρ' called resistivity or specific resistance of the material.

$$R = \frac{\rho l}{a}$$

5. What is the equivalent resistance of two parallel resistance

In case only two resistance are connected in parallel the equivalent resistance is given as.

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_1 + R_2}{R_1 R_2}$$

$$R_p = \frac{R_1 R_2}{R_1 + R_2}$$

* Show that the equivalent resistance of two resistances connected in parallel is the ratio of the product of these two resistances divided by sum of those two resistance values

Explain about voltage divider :-

The concept of voltage divider is very useful in analysing electric circuits. Considering the circuit which two resistances R_1 and R_2 are connected in series with voltage source V . The current I is given as,

$$I = \frac{V}{R_1 + R_2}$$

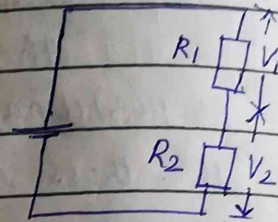
The voltage V_1 across resistance R_1 is given as,

$$V_1 = IR_1 = \frac{V}{R_1 + R_2} \cdot R_1$$

$$V_1 = V \left(\frac{R_1}{R_1 + R_2} \right)$$

Similarly, the voltage across R_2 is,

$$V_2 = V \left(\frac{R_2}{R_1 + R_2} \right)$$



Explain about current divider :-

* Two resistance R_1 and R_2 are connected in parallel

* same voltage across both resistance

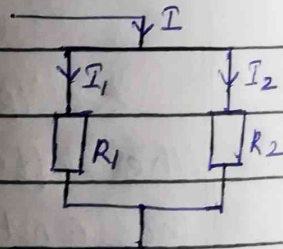
* $V = I_1 R_1$ $V = I_2 R_2$ $I_2 = \frac{I_1 R_1}{R_2}$

$$I = I_1 + I_2 = I_1 + I_1 \left(\frac{R_1}{R_2} \right) = I_1 \left(\frac{R_1 + R_2}{R_2} \right)$$

$$I_1 = I \left(\frac{R_2}{R_1 + R_2} \right)$$

Similarly the current through R_2 is given as,

$$I_2 = I \left(\frac{R_1}{R_1 + R_2} \right)$$



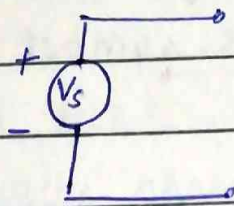
There are two types of energy sources

- 1) Voltage sources
- 2) Current sources

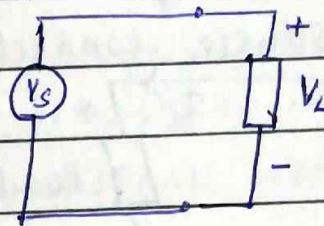
Ideal voltage sources:-

It is defined as the energy sources which gives constant voltage across its terminals irrespective of the current drawn through its terminals

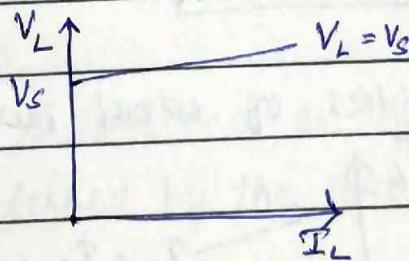
* Ideal voltage source symbol:-



* Ideal voltage source connected to the load.



* V-I characteristic of ideal voltage source.



* But practically every voltage source has small internal voltage resistance shown in series with voltage source and is represented by R_{se}

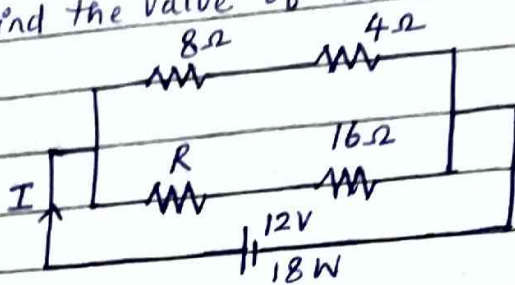
* Because of the R_{se} , voltage across terminals decreases with increase in the load current and it is given by expression

$$V_L = -(R_{se}) I_L + V_s = V_s - I_L R_{se}$$

* For ideal voltage source, $R_{se} = 0$ and for practical voltage source it is small as possible.

PROBLEMS

1. If the total power dissipated in the circuit shown is 18W, find the value of 'R' and its current.



given data:

$$P = 18W ; V = 12V$$

$$I = \frac{P}{V} = \frac{18}{12} = 1.5A$$

$$I_1 = \frac{V}{(8+4)}$$

$$= 12/12 = 1A$$

$$I_2 = I - I_1$$

$$= 1.5 - 1 = 0.5A$$

Voltage across 16Ω resistor is $V_{16} = I_2 \times 16$

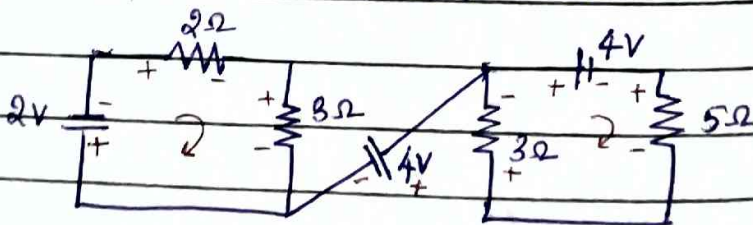
$$= 0.5 \times 16 = 8V$$

Voltage across 16Ω resistor is = 8V

Voltage across 'R' is $12 - 8 = 4V$

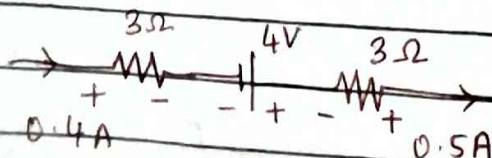
$$R = \frac{V}{I_2} = \frac{4}{0.5} = 8\Omega$$

2. what is the potential difference between the points X and Y in the network shown.



$$I_1 = \frac{2}{(2+3)} = 0.4A$$

$$I_2 = \frac{4}{(3+5)} = 0.5A$$

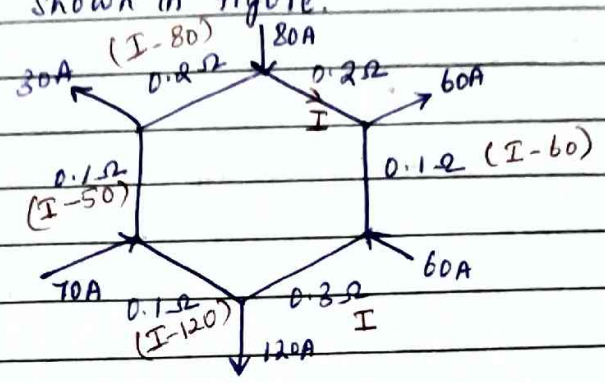


it shown

$$= (3 \times 0.4) + 4 - (3 \times 0.5)$$

$$= 3.7V$$

3. Find the values of current in all the branches of the networks shown in figure.



$$\Rightarrow 0.2(I) - 0.1(I-60) - 0.3I - 0.1(I-120) - 0.1(I-50) - 0.2(I-80) = 0$$

$$\Rightarrow -0.2I - 0.1I + 6.0 - 0.3I - 0.1I + 12 - 0.1I + 5 - 0.2I + 1.6 = 0$$

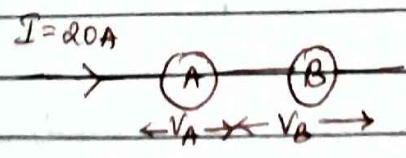
$$-I = -39$$

$$I = 39$$

8V

4. A current of 20A flows through two ammeters A and B in series. The potential difference across A is 0.2V and across B is 0.3V. Find how the same current will divide between A and B when they are in parallel.

Case 1:



$$V_A = 0.2V$$

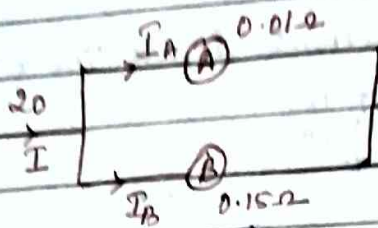
$$V_B = 0.3V$$

$$\text{Total } V = 0.5V$$

$$\therefore R_A = \frac{V}{I} = \frac{0.2}{20} = 0.01\Omega$$

$$R_B = \frac{V_B}{I} = \frac{0.3}{20} = 0.015 \Omega$$

Case II



using current division $I_A = I \times \frac{R_B}{R_A + R_B}$

$$= 20 \times \frac{0.15}{0.15 + 0.01} = 12A$$

$$I_B = I \times \frac{R_A}{R_A + R_B} =$$

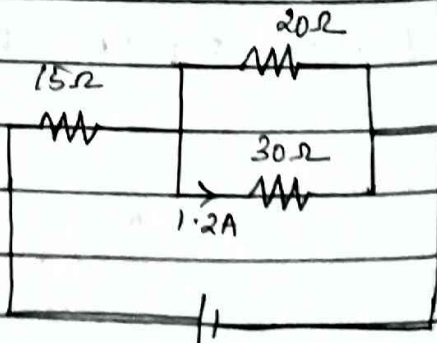
$$= 20 \times \frac{0.01}{0.15 + 0.01} = 8A$$

5. A circuit consists of 2 parallel resistors having resistances 20Ω and 30Ω respectively, connected in series with a 15Ω resistor. If the current through 30Ω resistor is $1.2A$, find (i) current in 20Ω and 15Ω resistor (ii) The voltage across the whole circuit (iii) Voltage across 15Ω resistor and 20Ω resistor (iv) total power consumed in the circuit.

Voltage across 30Ω is,

$$= IR$$

$$= 1.2 \times 30 = 36V$$



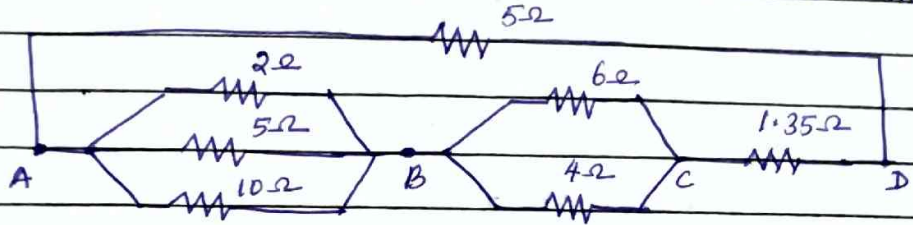
current in 20Ω is $V/R = 36/20$
 $= 1.8A$

Total current in the circuit is $= 1.8 + 1.2 = 3A$

Voltage in 15Ω is $= 3 \times 15 = 45V$

Total voltage is = $45 + 36 = 81V$
power in the circuit is = $VI = 81 \times 3 = 243W$.

6. Find the resistance of the circuit shown (R_{AD})



$$\frac{1}{R_{AB}} = \frac{1}{2} + \frac{1}{5} + \frac{1}{10}$$

$$= 0.5 + 0.2 + 0.1 = 0.8$$

$$\frac{1}{R_{AB}} = 0.8 \Rightarrow 1.25$$

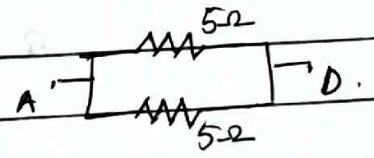
$$\therefore R_{AB} = 1.25\Omega$$

$$\frac{1}{R_{BC}} = \frac{1}{6} + \frac{1}{4} = 0.16 + 0.25$$

$$\frac{1}{R_{BC}} = 0.41$$

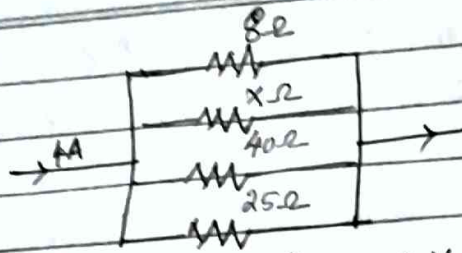
$$R_{BC} = 2.43\Omega$$

$$R_{AD} = R_{AB} + R_{BC} + R_{CD} = 1.25 + 2.43 + 1.35 = 5\Omega$$



$$R_{AD} = \frac{5 \times 5}{5 + 5} = \frac{25}{10} = 2.5\Omega$$

78. In the parallel arrangement of resistors shown the current flowing in the 8Ω resistor is $2.5A$. Find current in other resistor, resistor X, the equivalent resistance.



$$V = IR = 2.5 \times 8 = 20V$$

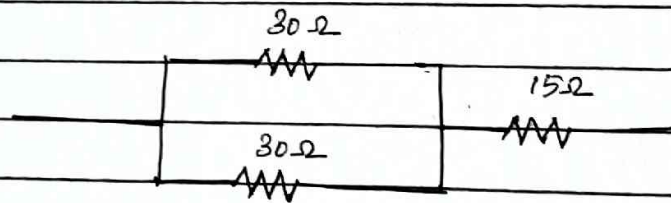
$$I_{40} = \frac{V}{R} = \frac{20}{40} = 0.5A$$

$$I_{25} = \frac{20}{25} = 0.8A$$

$$I_x = 4 - (2.5 + 0.8 + 0.5) = 0.2A$$

$$\text{Therefore } X(\Omega) = \frac{V}{I} = \frac{20}{0.2} = 100\Omega$$

8. A circuit of two parallel resistors having resistance of 20Ω and 30Ω respectively, connected in series with 15Ω . If the current through 15Ω resistor is $3A$, find (i) current in 20Ω and 30Ω resistor (ii) voltage across the whole circuit (iii) the power and power consumed in all resistors.



$$I_1 = 3 \times 30 / (30 + 20) = 1.8A$$

$$I_2 = 3 \times 20 / (30 + 20) = 1.2A$$

$$R_{eq} = 20 \times 30 / (20 + 30) = 27\Omega$$

$$V = IR = 3 \times 27 = 81V$$

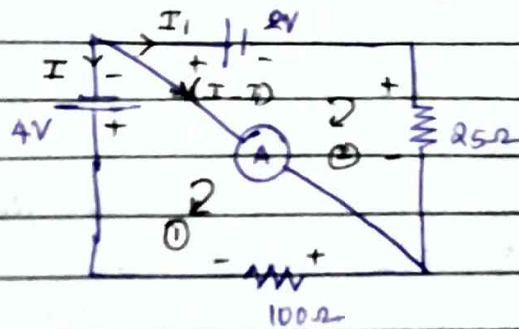
$$\text{Total power} = I^2 R = 3^2 \times 27 = 243W$$

$$P_{20} = (1.8)^2 \times 20 = 64.8W$$

$$P_{30} = (1.2)^2 \times 30 = 43.2W$$

$$P_{15} = (3)^2 \times 15 = 135W$$

9. In the network shown in figure, determine current flow in the ammeter A' having resistance of 10Ω



$$-100I - 4 - 10(I - I_1) = 0$$

$$110I - 10I_1 = -4 \quad \text{--- (1)}$$

$$-2 - 25I_1 + 10(I - I_1) = 0$$

$$10I - 35I_1 = 2 \quad \text{--- (2)}$$

Solving eq (1) + (2),

$$110I - 10I_1 = -4 \quad \text{(1) } \times 35$$

$$10I - 35I_1 = 2 \quad \text{--- (2) } \times 10$$

$$3850I - 350I_1 = -140$$

$$\Rightarrow 100I - 350I_1 = 20$$

$$\hline 3750I_1 = -160$$

$$I_1 = -0.042A$$

$$I_1 = -0.0693A$$

$$I - I_1 = 0.0263A$$

10. In the network shown find the current flowing in each branch using kirchoff's law.

Ac Fundamentals

1. Explain generation of sinusoidal voltage.

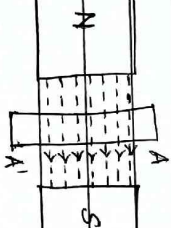
* It works on the principle of Faraday's law of electromagnetic induction

* The machine which use used to generate alternating quantities are called alternators.

* The main components of an alternator are stator and rotor.

* Stator is a stationary element and rotor is a rotating element.

* Alternating voltage can be generated in a stationary armature with rotating field structure or in a rotating armature with a stationary field.



* Consider an electromagnetic coil. Let this coil be rotating in anticlockwise direction with constant angular velocity ' ω ' in a uniform magnetic field between N and S poles.

* The electromotive force (emf) generated in a coil with respect to the uniform magnetic field.

* When AA' is perpendicular to the NS axis, the emf generated will be maximum in the coil and if it rotates 90° in the anticlockwise direction, the generated emf will be zero.

* Similarly, the emf generated is zero at 180° .

* This completes one-half cycle of the generated emf wave.

* In the next half cycle, the generated emf is negative, maximum at 270° from the initial position and zero

2.

at 360° . This completes another half-cycle of the emf wave.

2. Explain the following terms:-

Instantaneous Value:-

The value of voltage or current obtained ~~at any~~ at any instant of time is called instantaneous value.

$$e = E_m \sin \omega t$$

Wave form:-

The shape of the wave obtained when the instantaneous values of a periodic variable are plotted against time is called as waveform of the variable.

cycle:-

One complete set of positive and negative values of an alternating quantity is called as waveform of the variable.

Time period:-

The time taken by an alternating quantity to complete one cycle is called its time period or periodic time and is denoted by T

Frequency:-

The frequency of an alternating quantity is the number of cycles per second. It is denoted by f and is expressed in hertz. or cycles per second.

$$f = \frac{1}{T}$$

3. Average value and RMS value of alternating quantities

Average Value :-

- * The average value of a sine wave over one complete cycle is zero.
- * The average of a sine wave is defined over a half cycle.
- * The average value is the arithmetic mean of values at equal interval over a half-cycle.

$$\begin{aligned} \text{Vavg} &= \frac{1}{\pi} \int_0^{\pi} V_m \sin \omega t \, d\omega t = \frac{\text{Area under full cycle}}{\text{length of one cycle}} \\ &= \frac{V_m}{\pi} (-\cos \omega t)_0^{\pi} \\ &= \frac{2V_m}{\pi} = 0.637 V_m \end{aligned}$$

* It is defined as the value which is obtained by averaging all the instantaneous values over a period of half-cycle.

Root mean square (RMS) value or effective value :-

* It is defined as that steady value (dc) of quantity (Voltage or current) which when connected to a circuit for a given time produces the same heat effect produced by the alternating quantity flowing through the same circuit for the same time.

$$\begin{aligned} V &= V_m \sin \omega t \\ V_{\text{rms}} &= \sqrt{\frac{1}{\pi} \int_0^{\pi} (V_m \sin \omega t)^2 \, d\omega t} \\ V_{\text{rms}} &= \sqrt{\frac{1}{\pi} \int_0^{\pi} V_m^2 \sin^2 \omega t \, d\omega t} \end{aligned}$$

RMS value is defined on the basis of its heating effect.

Time

complete

cycle

alternates

1 cycle

$$\frac{V_m}{\sqrt{2}} \int_0^{\pi} \sin^2 \omega t \, d\omega t$$

$$\Rightarrow \sqrt{\frac{V_m^2}{2\pi} \int_0^{\pi} (1 - \cos 2\omega t) \, d\omega t} = \frac{V_m}{\sqrt{2}} \int_0^{\pi} (1 - \cos 2\omega t) \, d\omega t$$

$$\Rightarrow \sqrt{\frac{V_m^2}{2\pi} \left[\omega t - \frac{\sin 2\omega t}{2} \right]_0^{\pi}} = \frac{V_m}{\sqrt{2}} \left[\omega t - \frac{\sin 2\omega t}{2} \right]_0^{\pi}$$

$$V_{rms} = \frac{V_m}{\sqrt{2}} \Rightarrow \frac{V_m}{\sqrt{2}} \left[\pi - 0 \right] = \frac{V_m}{\sqrt{2}} \left[\pi - \frac{\sin 2\pi}{2} - \left(0 - \frac{\sin 0}{2} \right) \right]$$

4. Form factor

The form factor is defined as the ratio of an rms value to the average value of an alternating quantity

$$\text{Form factor} = \frac{\text{rms value}}{\text{Average value}} = \frac{V_{rms}}{V_{avg}}$$

5. Peak factor or crest factor

It is defined as the ratio of the maximum value to the rms value of an alternating quantity. Therefore,

$$\text{Peak factor} = \frac{\text{Maximum value}}{\text{rms value}} = \frac{V_m}{V_{rms}}$$

$$\text{Peak factor} = \frac{V_m}{V_m/\sqrt{2}}$$

$$= \sqrt{2} = 1.414$$

6. Phasor representation of alternating quantities

Phasors:-

* Phasor is a complex number that represents the amplitude and phase of a sinusoidal.
* The source voltage in time domain is given by

$$V(t) = V_m \sin \omega t$$

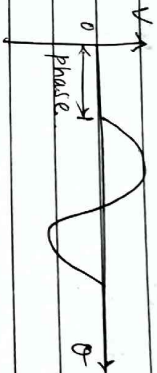
* The voltage in the frequency domain is given by $V = V_m \cos \omega t$. This representation is called phasor.

Vectors:-

- * It is a quantity having both magnitude and direction
- * A phasor has magnitude and phase (direction) and it behaves as vector.

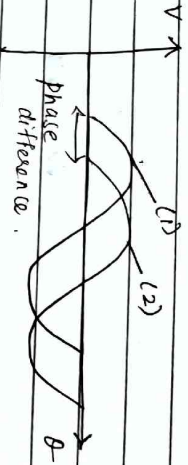
Phase:-

The phase of sine wave can be defined as the angular measurement of the wave that specifies the position of sine wave relative to a zero reference point.



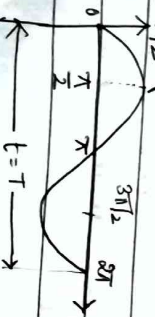
Phase difference:-

- * It is defined as the difference between the phases of two alternating quantities.
- * It is nothing but the angle difference between two phasors.
- * If the phase difference is 0° , we say both the quantities are in phase.



Phase angle:-

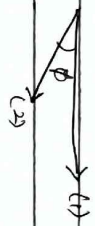
The phase of current at point P is $T/4$ s, where T is the time period expressed in terms of angle 90° or $\pi/2$.



Phasor diagram:-

* A graphical representation of phasor is known as the phasor diagram.

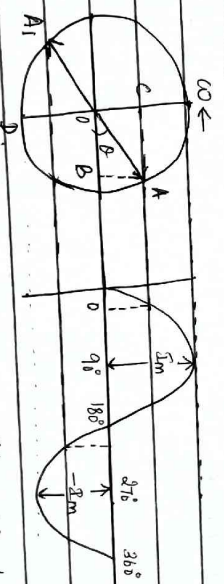
* When phasors are out of phase, from the phasor diagram, we can say whether the quantity is leading or lagging.



7. Phasor representation of alternating quantity by rotating vectors.

* The rotating vector representation of an alternating current, is observed that as time increases, the sinus rotates on a circle of radius I_m at an angular velocity ω in the counter clockwise direction.

* Let OA represents to scale, the maximum value of the current (ie) $OA = I_m$.



* OA is rotated by an angle θ from its original position when the current was passing through its zero value.

If AB and AC are its rectangular components then

$$OC = AB = OA \sin \theta$$

$$= I_m \sin \theta$$

* When $\theta = 90^\circ$ $OC = OA = I_m$

$\theta = 180^\circ$ current = 0

$\theta = 270^\circ$ $OA = -I_m$

$\theta = 370^\circ$ current = $-I_m$

* If f' is the frequency of the current wave in hertz, then ω rotates through f revolution

$\theta = \text{Angular Velocity} \times \text{Time}$

$= \omega t$
 $\theta = 2\pi ft$

8. The alternating current is given by $i = 20 \sin 600t$ amperes. Find (a) the frequency (b) the peak value of current and (c) the time taken from $t=0$ for the current to reach a value of 10A.

$i = I_m \sin(\omega t + \phi)$
 $i = 20 \sin 600t$

(a) $\omega = 600 \text{ rad/s}$, since $\omega = 2\pi f$
 $f = \frac{\omega}{2\pi} = \frac{600}{2\pi} = 95.5 \text{ Hz}$

(b) The peak value of current, $I_m = 20 \text{ A}$.
 (c) The time for the current to reach 10A is given by.

$i = 20 \sin 600t$
 $10 = 20 \sin 600t$
 $600t = \sin^{-1}\left(\frac{10}{20}\right) = 30^\circ = \frac{30 \times \pi}{180} \text{ rad} = 0.5236 \text{ rad}$
 $t_1 = \frac{0.523}{600} = 0.000872 \text{ s} = 873 \mu\text{s}$

9. An alternating current varying sinusoidally with a frequency of 50Hz has rms value of 20A. At what time, measured from negative maximum value, instantaneous current will be $10\sqrt{2} \text{ A}$?

Peak value of the current is $I_m = 20\sqrt{2} \text{ A}$.

This is just a sine wave with phase lag $\phi = \pi/2$

$i = 20\sqrt{2} \sin(100\pi t - \pi/2)$
 $10\sqrt{2} = 20\sqrt{2} \sin(100\pi t - \pi/2)$
 $\sin^{-1}\left(\frac{1}{2}\right) = 100\pi t - \pi/2$

$$100\pi t = \sin^{-1}(1/2) + \pi/2$$
$$t = \frac{\pi/2 + \pi/2}{100\pi} = 0.00667 \text{ s} = 6.67 \text{ ms}$$

Q. Given the sinusoid $10 \sin(\omega t - 40^\circ)$, calculate its amplitude, phase, angular frequency, period and frequency.

$$10 \sin(\omega t - 40^\circ)$$

$$\text{Amplitude} = 10$$

$$\text{Phase} = -40^\circ$$

$$\text{Angular Velocity} = 6.28 \text{ rad/s}$$

$$\text{Period} = 1 \text{ s}$$

$$\text{Frequency} = 1 \text{ Hz}$$

rad