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ENGINEERING MATHS I (ACN)
(18MAT11)
MODULE - 5

A set of mn Elements written in a array of m rows and n columns within the square brackets is called a matrix of order $m \times n$

Example: $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \vdots & & & & \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}_{m \times n}$

If $m=n$ then it is called square matrix

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}_{m \times n}$$

A matrix of order $1 \times n$ is Row matrix. A matrix of order $m \times 1$ is column matrix.

$A = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}_{1 \times n}$ $\quad A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}_{m \times 1}$

A Square is said to be diagonal matrix if Every Element of a matrix other than the principle diagonal are zero.

$$A = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 10 \end{bmatrix}$$

A matrix in which all the diagonal Elements are Same and other terms are zero scalar matrix.

$$A = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

A Scalar matrix in which each diagonal elements are equal to unity i.e., 1 is called Identity matrix (or) unit matrix.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(1) $A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & -6 & -7 \end{bmatrix}$ $\quad$ $\text{adj } A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}^{T}$

$$\text{adj } A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & -6 & -7 \end{bmatrix}^{T}$$

$$\begin{bmatrix} 1(-28+30) & -0(-21-0) & -1(-18-0) \\ -3(0-6) & +4(-7+0) & -5(-6-0) \\ +0(0+4) & +6(5+3) & -7(4-0) \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 18 \\ 9 & -28 & 30 \\ 4 & 48 & -28 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & -6 & -7 \end{vmatrix} = 1(-28)+30-0(-21-0)-1(-18-0)$$

$$= 20$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{20} \begin{bmatrix} 2 & 9 & 4 \\ 6 & -28 & 48 \\ 18 & 30 & -28 \end{bmatrix}$$

Find the rank of the matrix by row transformation

(i)

$$A = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$$

$R_3 \rightarrow R_3 - R_1$

$$\begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 0 & 1 & 1 & 3 \end{bmatrix}$$

$R_2 \rightarrow R_2 - 2R_1$

$$\begin{bmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & -1 & -3 \\ 0 & 1 & 1 & 3 \end{bmatrix}$$

$R_3 \rightarrow R_3 + R_2$

$$\begin{bmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & -1 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}_{3\times 4}$$

Rank $\rho(A) = 2$

2. $$A = \begin{bmatrix} 0 & 2 & 3 & 4 \\ 2 & 3 & 5 & 4 \\ 4 & 8 & 13 & 12 \end{bmatrix}$$

$R_2 \longleftrightarrow R_1$

$$\begin{bmatrix} 2 & 3 & 5 & 4 \\ 0 & 2 & 3 & 4 \\ 4 & 8 & 13 & 12 \end{bmatrix}$$

$R_3 \rightarrow R_3 - 2R$

$$\begin{bmatrix} 2 & 3 & 5 & 4 \\ 0 & 2 & 3 & 4 \\ 0 & 2 & 3 & 4 \end{bmatrix}$$

$R_3 \rightarrow R_3 - R_2$

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$$\begin{bmatrix} 2 & 3 & 5 & 4 \\ 0 & 2 & 3 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R \rightarrow \frac{1}{2} R_1$ $\begin{bmatrix} 1 & 3/2 & 5/2 & 2 \\ 0 & 2 & 3 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$\rho(A) = 2$

$\Rightarrow$

$$A = \begin{bmatrix} 2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

$R_1 \leftrightarrow R_2$ $\qquad$ $R_2 \rightarrow R_2 - 2R_1$

$$\begin{bmatrix} 1 & 2 & 3 & -1 \\ 2 & -1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & -5 & -9 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

$R_3 \rightarrow R_3 - R_1$ $\qquad$ $R_2 \leftrightarrow R_4$

$$\begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & -5 & -9 & 1 \\ 0 & -2 & -2 & 2 \\ 0 & 1 & 1 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & 1 & 1 & -1 \\ 0 & -2 & -2 & 2 \\ 0 & -5 & -9 & 1 \end{bmatrix}$$

$R_4 \rightarrow R_4 + 5R_2$ $\qquad$ $R_3 \rightarrow R_3 + 2R_2$

$$\begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & 1 & 1 & -1 \\ 0 & -2 & -2 & 2 \\ 0 & 0 & -4 & -4 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -4 & -4 \end{bmatrix}$$

$R_3 \leftrightarrow R_4$

$$\begin{pmatrix} 1 & 2 & 3 & -1 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & -4 & -4 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(4)

$$\begin{bmatrix} 4 & 0 & 2 & 1 \\ 2 & 1 & 3 & 4 \\ 2 & 3 & 4 & 7 \\ 2 & 3 & 1 & 4 \end{bmatrix}$$

$R_2 \leftrightarrow R_1$

$$\begin{bmatrix} 2 & 1 & 3 & 4 \\ 4 & 0 & 2 & 1 \\ 2 & 3 & 4 & 7 \\ 2 & 3 & 1 & 4 \end{bmatrix}$$

$R_3 \rightarrow R_3 - R_4$

$$\begin{bmatrix} 2 & 1 & 3 & 4 \\ 4 & 0 & 2 & 1 \\ 0 & 0 & 3 & 9 \\ 2 & 3 & 1 & 4 \end{bmatrix}$$

$R_4 \rightarrow R_4 - R_1$

$$\begin{bmatrix} 2 & 1 & 3 & 4 \\ 4 & 0 & 2 & 1 \\ 0 & 0 & 3 & 3 \\ 0 & 2 & -2 & 0 \end{bmatrix}$$

$R_4 \rightarrow R_4 + R_2$

$$\begin{bmatrix} 2 & 1 & 3 & 4 \\ 4 & 0 & 2 & 1 \\ 0 & 0 & 3 & 3 \\ 0 & 2 & 0 & 1 \end{bmatrix}$$

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(5) $\begin{pmatrix} 1 & 2 & 1 \\ 2 & 3 & 3 \\ 3 & 5 & 4 \\ 2 & 1 & 5 \end{pmatrix}$

$R_2 \to R_2 - R_4 \quad R_3 \to R_3 - 3R_1 \quad R_4 \to R_4 - 2R_1$

$\begin{pmatrix} 1 & 2 & 1 \\ 0 & 2 & -2 \\ 0 & -1 & -1 \\ 0 & -3 & 3 \end{pmatrix}$

$R_3 \to 2R_3 + R_2 \quad , R_4 \to 1/3 R_4 \quad , R_{\cancel{4}} \to 2R_4 + R_2$

$\begin{pmatrix} 1 & 2 & 1 \\ 0 & 2 & -2 \\ 0 & 0 & -4 \\ 0 & -1 & 1 \end{pmatrix} \qquad \begin{pmatrix} 1 & 2 & 1 \\ 0 & 2 & -2 \\ 0 & 0 & -4 \\ 0 & 0 & 0 \end{pmatrix}$

~~$\rho(A) = 3$~~ $\quad \rho(A) = 3$

(6) $\begin{bmatrix} 1 & 3 & 4 & 5 \\ 3 & 2 & 5 & 2 \\ 2 & -1 & 1 & -3 \end{bmatrix}$

$R_3 \to R_3 - R_2 \qquad R_3 \to R_3 + R_1 \quad R_2 \to R_2 - 3R_3$

$\begin{bmatrix} 1 & 3 & 4 & 5 \\ 3 & 2 & 5 & 2 \\ -1 & -3 & -4 & -5 \end{bmatrix} \qquad \begin{bmatrix} 1 & 3 & 4 & 5 \\ 0 & -7 & -7 & -13 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$\rho(A) = 2$

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Find the value of k such that matrix is A they may have rank equal to
(i) 3 (ii) 2

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & k \\ 1 & 4 & 10 & k^2 \end{bmatrix}$$

$R_3 \to R_3 - R_2$, $R_2 \to R_2 - R_1$, $R_3 \to R_3 - 2R_2$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & (k-1) \\ 0 & 2 & 6 & (k^2-k) \end{bmatrix} \quad \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & k-1 \\ 0 & 0 & 0 & (k^2-k)-2(k-1) \end{bmatrix}$$

$k^2 - k - 2k + 2 - k^2 - 3k + 2$

1) $k^2 - 3k + 2 \neq 0$

$k^2 - 2k - k + 2 = 0$

$k(k-2) - 1(k-2) = 0$

$(k-1)(k-2) = 0$

$k = 1, k = 2$

In this case $k \neq 1, k = 2$

2) $k^2 - 3k + 2 = 0$

$k = 1, k = 2$

Column transformation

$$A = \begin{bmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

$R_1 \leftrightarrow R_2$ $\qquad$ $R_3 \to R_3 - R_1$, $R_2 \to R_2 + 2R$

$$\begin{bmatrix} 1 & 2 & 3 & 1 \\ -2 & -1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & 3 & 3 & 1 \\ 0 & -2 & -2 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

$R_4 \to 2R_4 + R_3$ $\qquad$ $C_2 \to C_2 - C_3$

$$\begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & 3 & 3 & 1 \\ 0 & -2 & 2 & 1 \\ 0 & 0 & 0 & 2 \end{bmatrix} \quad \begin{bmatrix} 1 & -1 & 3 & 1 \\ 0 & 0 & 3 & 1 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

$$\rho(A) = 4$$

Gauss Elimination Method

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$$
$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$$
$$Ax = B$$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad B = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

Solve the following system of Equation by gauss Elimination method:-

(1)
$$x + y + z = 9$$
$$x - 2y + 3z = 8$$
$$2x + y - z = 3$$

$$A : B \quad \begin{bmatrix} 1 & 1 & 1 & : & 9 \\ 1 & -2 & 3 & : & 8 \\ 2 & 1 & -1 & : & 3 \end{bmatrix}$$

$R_3 \to R_3 - 2R_1$, $R_2 - R_2 - R_1$ $\qquad R_3 \to 3R_3 - R_2$

$$\begin{bmatrix} 1 & 1 & 1 & : & 9 \\ 0 & -3 & 2 & : & -1 \\ 0 & -1 & -3 & : & -15 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 & 1 & : & 9 \\ 0 & -3 & 2 & : & -1 \\ 0 & 0 & -11 & : & -44 \end{bmatrix}$$

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$$x+y+z=9$$
$$-3y+2z=-1$$
$$-11z=-44$$

$z=\frac{-44}{11} \Rightarrow z=4$

$-3y+2(4)=-1$

$-3y=-8-1$

$y=\frac{-9}{-3}$

$y=3$

$x+3+4=9$

$x=2$

(2)

$$2x+y+4z=12$$
$$4x+11y-z=33$$
$$8x-3y+2z=20$$

$$[A:B]=\begin{bmatrix}2 & 1 & 4 & : & 12\\ 4 & 11 & -1 & : & 33\\ 8 & -3 & 2 & : & 20\end{bmatrix}$$

$R_3 \to R_3-2R_2$, $R_2 \to R_2-2R_1$ $\quad R_3 \to 9R_3+25R_2$, $R_2 \to 1/9$

$$\begin{bmatrix}2 & 1 & 4 & : & 12\\ 0 & 9 & -9 & : & 9\\ 0 & -25 & 4 & : & -46\end{bmatrix} \qquad \begin{bmatrix}2 & 1 & 4 & : & 12\\ 0 & 1 & -1 & : & 1\\ 0 & 0 & -189 & : & -189\end{bmatrix}$$

$2x+y+4z=12$

$y-z=1$

$-189z=-189$

$z=\cancel{189}$

$z=1$

$y-z=1$

$y=1+1$

$y=2$

$2x+2+(4)(1)=12$

$2x=12-6$

$x=3$

(3) $5x_1 + x_2 + x_3 + x_4 = 4$

$x_1 + 7x_2 + x_3 + x_4 = 12$

$x_1 + x_2 + 6x_3 + x_4 = -5$

$x_1 + x_2 + x_3 + 4x_4 = -6$

$$[A:B] = \left[\begin{array}{cccc:c} 5 & 1 & 1 & 1 & 4 \\ 1 & 7 & 1 & 1 & 12 \\ 1 & 1 & 6 & 1 & -5 \\ 1 & 1 & 1 & 4 & -6 \end{array}\right]$$

$R_4 \to 5R_4 - R_1$, $R_3 \to R_3 - R_2$, $R_2 \to R_2 - R_4$

$$\left[\begin{array}{cccc:c} 5 & 1 & 1 & 1 & 4 \\ 0 & 6 & 0 & -3 & 18 \\ 0 & -6 & 5 & 0 & -17 \\ 0 & 4 & 4 & 19 & -34 \end{array}\right]$$

$R_3 \to R_3 + R_2$, $R_2 \to 1/3 R_2$ $\qquad$ $R_4 \to R_4 - 2R_2$

$$\left[\begin{array}{cccc:c} 5 & 1 & 1 & 1 & 4 \\ 0 & 2 & 0 & -1 & 6 \\ 0 & 0 & 5 & -3 & 1 \\ 0 & 4 & 4 & 19 & -34 \end{array}\right] \qquad \left[\begin{array}{cccc:c} 5 & 1 & 1 & 1 & 4 \\ 0 & 2 & 0 & -1 & 6 \\ 0 & 0 & 5 & -3 & 1 \\ 0 & 0 & 4 & 21 & -46 \end{array}\right]$$

$R_4 \to 5R_4 - 4R_3$

$$\left[\begin{array}{cccc:c} 5 & 1 & 1 & 1 & 4 \\ 0 & 2 & 0 & -1 & 6 \\ 0 & 0 & 5 & -3 & 1 \\ 0 & 0 & 0 & 117 & -234 \end{array}\right]$$

$5x_1 + x_2 + x_3 + x_4 = 4$

$2x_2 - x_4 = 6$

$5x_3 - 3x_4 = 1$

$117x_4 = -234$

$$x_4 = \frac{-234}{117} \qquad x_4 = -2$$

$x_2 = 2$

$x_1 = 1$

$x_3 = -1$

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Gauss Jordan Method

①
$x+y+z=9$
$x-2y+3z=8$
$2x+y-z=3$

$$[A:B] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 1 & -2 & 3 & 8 \\ 2 & 1 & -1 & 3 \end{array}\right]$$

$R_3 \to R_3 - 2R_1$, $R_2 \to R_2 - R_1$ $\qquad$ $R_3 \to 3R_3 - R_2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & -3 & 2 & -1 \\ 0 & -1 & -3 & -15 \end{array}\right] \qquad \left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & -3 & 2 & -1 \\ 0 & 0 & -11 & -44 \end{array}\right]$$

$R_1 \to 3R_1 + R_2$, $R_3 \to R_3/_{-11}$ $\qquad$ $R_1 \to R_1 - 5R_3$, $R_2 \to R_2 - 2R_3$

$$\left[\begin{array}{ccc|c} 3 & 0 & 5 & 26 \\ 0 & -3 & 2 & -1 \\ 0 & 0 & 1 & 4 \end{array}\right] \qquad \left[\begin{array}{ccc|c} 3 & 0 & 0 & 6 \\ 0 & -3 & 0 & -9 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

$3x = 6$ $\qquad$ $-3y = -9$ $\qquad$ $z = 4$
$x = 2$ $\qquad$ $y = 3$

$3(9)-1$
$27-1$
26

②
$2x+5y+7z=52$
$2x+y-z=0$
$x+y+z=9$

$$A:B = \left[\begin{array}{ccc|c} 2 & 5 & 7 & 52 \\ 2 & 1 & -1 & 0 \\ 1 & 1 & 1 & 9 \end{array}\right]$$

$R_3 \to R_1$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 2 & 1 & -1 & 0 \\ 2 & 5 & 7 & 52 \end{array}\right]$$

$R_3 \to R_3 - R_2$, $R_2 \to R_2 - 2R_1$, $R_3 \to R_3 + 4R_2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & -1 & -3 & -18 \\ 0 & 4 & 8 & 52 \end{array}\right] \qquad \left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & -1 & -3 & -18 \\ 0 & 0 & -4 & -20 \end{array}\right]$$

$R_1 \to R_1 + R_2$

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & -9 \\ 0 & -1 & -3 & -18 \\ 0 & 0 & -4 & -20 \end{array}\right]$$

$R_1 \to 2R_1 - R_3$

$$\left[\begin{array}{ccc|c} 2 & 0 & 0 & 2 \\ 0 & -1 & -3 & -18 \\ 0 & 0 & -4 & -20 \end{array}\right]$$

$R_2 \to 4R_2 - 3R_3$

$$\left[\begin{array}{ccc|c} 2 & 0 & 0 & 2 \\ 0 & -4 & 0 & -12 \\ 0 & 0 & -4 & -20 \end{array}\right]$$

$$2x = 2$$

$$x = 1$$

$$-4y = -12$$

$$y = 3$$

$$-4z = -20$$

$$z = 5$$

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Solve the following System of Equations by Gauss seidel method:-

(1) ~~10x+y = 12~~ $10x + y + z = 12$

$x + 10y + z = 12$

$x + y + 10z = 12$

$|10| > |1| + |1|$ ✓

$|10| > |1| + |1|$ ✓

$|10| > |1| + |1|$ ✓

$$x = \frac{1}{10}(12 - y - z)$$

$$y = \frac{1}{10}(12 - x - z)$$

$$z = \frac{1}{10}(12 - x - y)$$

$x = 0, \quad y = 0, \quad z = 0$

$$x = \frac{1}{10}(12 - 0 - 0)$$

$$\boxed{x^{(1)} = 1.2}$$

$$y = \frac{1}{10}(12 - 1.2 - 0)$$

$$\boxed{y^{(1)} = 1.08}$$

$$z = \frac{1}{10}(12 - 1.2 - 1.08)$$

$$\boxed{z^{(1)} = 0.972}$$

$$x = \frac{1}{10}(12 - 1.08 - 0.972)$$

$$\boxed{x^{(2)} = 0.9948}$$

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$$y^{(2)} = \frac{1}{10}(12 - 0.9948 - 0.972)$$

$$\boxed{y^{(2)} = 1.0033}$$

$$z^{(2)} = \frac{1}{10}(12 - 0.9948 - 1.0033)$$

$$\boxed{z^{(2)} = 1.0001}$$

$$x^{(3)} = \frac{1}{10}(12 - 1.0033 - 1.0001)$$

$$\boxed{x^{(3)} = 0.99966}$$

$$y^{(3)} = \frac{1}{10}(12 - 0.99966 - 1.0001) \Rightarrow y = 1.000024$$

$$z^{(3)} = \frac{1}{10}(12 - 0.99966 - 1.00002)$$

$$\boxed{z^{(3)} = 1.00003}$$

$$\boxed{y = 1}$$

$$x = 0.99 \approx 1 \qquad z = 1$$

Solve the foll sy of Equ by Gauss Seidel method

(2)

$$x + y + 54z = 110$$
$$27x + 6y - z = 85$$
$$6x + 15y + 2z = 72$$

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$$|27| > |6| + |-1| \Rightarrow x = 1/27\,(85 - 6y + z)$$
$$|15| > |6| + |2| \Rightarrow y = 1/15\,(72 - 6x - 2z)$$
$$|54| > |1| + |1| \Rightarrow z = 1/54\,(110 - x - y)$$

$$x = 0,\ y = 0,\ z = 0$$

$3.5720 \rightarrow y^{(2)}$
1.9258

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$$x^{(1)} = \frac{1}{27}(85 - 0 + 0) \Rightarrow x^{(1)} = 3.1481$$

$$y^{(1)} = \frac{1}{15}(72 - 6(3.1481) - 2(0)] \Rightarrow y^{(1)} = 3.5407$$

$$z^{(1)} = \frac{1}{54}(110 - 3.1481 - 3.5407)$$

$$\boxed{z^{(1)} = 1.9131}$$

$$\cancel{x}^{(2)} = \frac{1}{27}(85 - 6(3.5407) + 1.9131) \Rightarrow x^{(2)} = 2.4321$$

$$y^{(2)} = \frac{1}{15}(72 - 6(2.43218) - 2(1.91317)) = 3.57204$$

$$z^{(2)} = \frac{1}{54}[110 - 2.43218 \cdot 357204] = 1.9258$$

$$x^{(3)} = \frac{1}{27}[85 - 6(3.5720) + 1.9258] = 2.4256$$

$$y^{(3)} = \frac{1}{15}[72 - 6(2.4256) - 2(1.9258)] = 3.5729$$

$$z^{(3)} = \frac{1}{54}[110 - 2.4256 - 3.5729] = 1.9259$$

x ~~= 2.43~~ $x = 2.42$.
$y = 3.57$
$z = 1.92$

$\Rightarrow$

$$5x + 2y + z = 12$$
$$x + 4y + 2z = 15$$
$$x + 2y + 5z = 20$$

$$|5| > |2| + |1| \checkmark$$
$$|4| > |2| + |1| \checkmark$$
$$|5| > |2| + |1| \checkmark$$

$$x = \frac{1}{5}(12 - 2y - z)$$

$$y = \frac{1}{4}(15 - x - 2z)$$

$$z = \frac{1}{5}(20 - x - 2y)$$

$$x = 0, \quad y = 0, \quad z = 0$$

$$x^{(1)} = \frac{1}{5}(12 - 0 - 0)$$

$$= 2.4$$

$$y^{(1)} = \frac{1}{4}(15 - 2.4 - 0)$$

$$= 3.75$$

$$z^{(1)} = \frac{1}{5}(20 - x - 2(3.75))$$

$$= 2.5$$

$$x^{(2)} = \frac{1}{5}(12 - 2(3.15) - 2.26) = 0.688$$

$$y^{(2)} = \frac{1}{4}(15 - 0.688 - 2(2.26) = 0.2448$$

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$$z^{(2)} = \frac{1}{5}\left(20 - 0.688 - 2(2.488\right) = 2.8832$$

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Given $X' = x_1, x_2, x_3, x_4$ $\begin{bmatrix} x_1 & x_2 \\ x_3 & x_4 \end{bmatrix}$ $B' = \begin{bmatrix} 3 & 15 \\ 27 & -9 \end{bmatrix}$

& $A' = \begin{bmatrix} 10 & -2 & -1 & -1 \\ -2 & 10 & -1 & -1 \\ -1 & -1 & 10 & -2 \\ -1 & -1 & -2 & 10 \end{bmatrix}$

Write down the system of eqn $AX = B$. Solve the system by Gauss Seidal method by taking 0.9, 1.9, 2.9, 0 as initial approximation for the Solution.

Soln:-

$$A = \begin{bmatrix} 10x_1 & -2x_2 & -x_3 & -x_4 \\ -2x_1 & +10x_2 & -x_3 & -x_4 \\ -x_1 & -x_2 & +10x_3 & -2x_4 \\ -x_1 & -x_2 & -2x_3 & +10x_4 \end{bmatrix} \begin{matrix} = 3 \\ = 15 \\ = 27 \\ = -9 \end{matrix}$$

$$x_1 = \frac{1}{10}(3 + 2x_2 + x_3 + x_4)$$

$$x_2 = \frac{1}{10}(15 + 2x_1 + x_3 + x_4)$$

$$x_3 = \frac{1}{10}(27 + x_1 + x_2 + 2x_4)$$

$$x_4 = \frac{-1}{10}(9 - x_1 - x_2 - 2x_3)$$

$$x_1^{(1)} = 0.97$$

$$x_2^{(1)} = 1.984$$ 2.17

0.4

$$x_3^{(1)} = 2.99$$

$$x_4^{(1)} = 0.00552$$ - 0.0C4

$$x_1^{(2)} = \frac{1}{10}(3 + 2x_2 + x_3 + x_4) = 0.9958$$

$$x_2^{(2)} = \frac{1}{10}(15 + 2x_1 + x_3 + x_4) = 1.9987$$

$$x_3^{(2)} = \frac{1}{10}(27 + x_1 + x_2 + 2x_4) = 2.9954$$

$$x_4^{(2)} = \frac{1}{10}(9 - x_1 - x_2 - 2x_3) = 0.00093$$

$x_1 = 0.99 \approx 1$
$x_2 = 1.9 \approx 2$
$x_3 = 2.9 \approx 3$
$x_4 = 0.005 = 0$

Linear transformation

If $\alpha = x\cos\theta - y\sin\theta$ and $\beta = x\sin\theta + y\cos\theta$ write the matrix A of this transformation and find the value of x (or) Find inverse transformation.

$\alpha = x\cos\theta - y\sin\theta$
$\beta = x\sin\theta + y\cos\theta$

$$AX = Y$$

$$\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$X = A^{-1}Y$$

$$A^{-1} = \frac{1}{|A|} \text{Adj } A$$

$$|A| = \cos^2\theta + \sin^2\theta$$

$$|A| = 1$$

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$$\text{Adj} A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{Adj } A$$

$$= \frac{1}{1} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$X = A^{-1} Y$$

$$X = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$

(2) Find the inverse transformation of the following linear

$$\begin{pmatrix} 19 & -9 & 2 \\ -4 & 2 & -1 \\ -2 & 1 & 0 \end{pmatrix}$$

(1) $y_1 = x_1 + 2x_2 + 5x_3$
$y_2 = 2x_1 + 4x_2 + 11x_3$
$y_3 = -x_2 + 2x_3$

$Y = AX$

$x = A^{-1} Y$

$$A = \begin{bmatrix} 1 & 2 & 5 \\ 2 & 4 & 11 \\ 0 & -1 & 2 \end{bmatrix}$$

$|A| = 1$

$8+11 \quad 4-0 \quad -2-0$

$19 \quad 4 \quad -2$

$1(8+11) - 2(4-0) + 5(-2-0)$

$19 - 8 + 10$

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$$A^{-1} = \begin{bmatrix} 19 & -9 & 2 \\ -4 & 2 & -1 \\ -2 & 1 & 0 \end{bmatrix}$$

$$X = A^{-1}Y$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 19 & -9 & 2 \\ -4 & 2 & -1 \\ -2 & 1 & 0 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$$x_1 = 19y_1 - 9y_2 + 2y_3$$
$$x_2 = -4y_1 + 2y_2 - y_3$$
$$x_3 = -2y_1 + y_2$$

(3) Given the linear transformation $y_1 = 5x_1 + 3x_2 + 3x_3$, $y_2 = 3x_1 + 2x_2 - 2x_3$, $y_3 = 2x_1 - x_2 + 2x_3$ and $z_1 = 4x_1 + 2x_3$, $z_2 = x_2 + 4x_3$, $z_3 = 5x_3$. Establish the linear transformation to $z_1, z_2 \& z_3$ from y_1, y_2, y_3 by matrix approach.

$$A = \begin{bmatrix} 5 & 3 & 3 \\ 3 & 2 & -2 \\ 2 & -1 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} 4 & 0 & 2 \\ 0 & 1 & 4 \\ 0 & 0 & 5 \end{bmatrix}$$

$$|B| = 4(5-0) - 0(\ ) + 2(0-0)$$
$$|B| = 20$$

$$Y = AX$$
$$Z = BX$$
$$X = B^{-1}Z$$
$$Y = A(B^{-1}Z)$$
$$= (AB^{-1})Z$$

$$B^{-1} = \begin{bmatrix} 5 & 0 & -2 \\ 0 & 20 & -16 \\ 0 & 0 & 4 \end{bmatrix}$$

$$X = B^{-1}Y$$

$$A =$$
$$B^{-1} =$$

$2(5-0)$
$2(0-0)$
$0($
$4(5-0)$

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~~AB~~ A×B

$$\begin{bmatrix} 5 & 3 & 3 \\ 3 & 2 & -2 \\ 2 & -1 & 2 \end{bmatrix} \begin{bmatrix} 5 & 0 & -2 \\ 0 & 20 & -16 \\ 0 & 0 & 4 \end{bmatrix}$$

$\frac{16\times3}{48}$

$$\begin{bmatrix} 25+0+0 & 0+60+0 & -10-48+12 \\ 15+0+0 & 0+40+0 & -6+32+8 \\ 10+0+0 & 0-20+0 & -4+16+8 \end{bmatrix}$$

6   14
-8   32
-46
10 -20 20

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 25 & 60 & -46 \\ 15 & 40 & -46 \\ 10 & -20 & 20 \end{bmatrix}$$

$\frac{25}{20} \frac{5}{4}$

$$\frac{1}{20} = \begin{bmatrix} 25 & 60 & -46 \\ 15 & 40 & -46 \\ 10 & -20 & 20 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}$$

$$y_1 = \frac{25}{20} z_1 + \frac{60}{20} z_2 + \frac{-46}{20} z_3$$

$$y_2 = \frac{15}{20} z_1 + \frac{40}{20} z_2 + \frac{-46}{20} z_3$$

$$y_3 = \frac{10}{20} z_1 + \frac{-20}{20} z_2 + \frac{20}{20} z_3$$

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Find all the Eigen values and the corresponding Eigen vectors of the matrix

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{vmatrix} = 0$$

$$(8-\lambda)\left[(7-\lambda)(3-\lambda)-16\right] + 6\left(-6(3-\lambda)+8\right) + 2\left(24-2(7-\lambda)\right) = 0$$

$$(8-\lambda)\left[21-10\lambda+\lambda^2-16\right] + 6(-18+6\lambda+8) + 2(24-14+2\lambda) = 0$$

$$(8-\lambda)(\lambda^2-10\lambda+5) + 36\lambda - 60 + 20 + 4\lambda = 0$$

$$8\lambda^2 - 80\lambda + 40 - \lambda^3 + 10\lambda^2 - 5\lambda + 36\lambda - 60 + 20 + 4\lambda = 0$$

$$-\lambda^3 + 18\lambda^2 - 45\lambda = 0$$

$$\lambda^3 - 18\lambda^2 + 45\lambda = 0$$

$$\lambda(\lambda^2 - 18\lambda + 45) = 0$$

$$\lambda = 0, \quad \lambda^2 - 18\lambda + 45 = 0$$

$$\lambda^2 - 15\lambda - 3\lambda + 45 = 0$$

$$\lambda = 0, \quad \lambda = 3, \quad \lambda = 15$$

$$\left.\begin{aligned} (8-\lambda)x - 6y + 2z &= 0 \\ -6x + (7-\lambda)y - 4z &= 0 \\ 2x - 4y + (3-\lambda)z &= 0 \end{aligned}\right\} \rightarrow (1)$$

Case 1) $\rightarrow \lambda = 0$

$$8x - 6y + 2z = 0 \rightarrow (2)$$

$$-6x + 7y - 4z = 0 \rightarrow (3)$$

$$2x - 4y + 3z = 0 \rightarrow (4)$$

Apply Rule of cross x^n for (2) & (3)

$$\frac{x}{\begin{vmatrix} -6 & 2 \\ 7 & -4 \end{vmatrix}} = \frac{-y}{\begin{vmatrix} 8 & 2 \\ -6 & -4 \end{vmatrix}} = \frac{z}{\begin{vmatrix} 8 & -6 \\ -6 & 7 \end{vmatrix}}$$

20
12

20
36

$$\frac{x}{\cancel{10}_{1}} = \frac{-y}{\cancel{-20}_{2}} = \frac{z}{\cancel{20}_{2}}$$

$$\Rightarrow \frac{x}{1} = \frac{y}{2} = \frac{z}{2}$$

$$X_1 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} \text{ (or) } X_1 = (1, 2, 2)$$

Case 2 : $\longrightarrow \lambda = 3$

$$5x - 6y + 2z = 0$$
$$-6x + 4y - 4z = 0$$
$$2x - 4y + 0z = 0$$

$$\frac{x}{\begin{vmatrix} -6 & 2 \\ 4 & -4 \end{vmatrix}} = \frac{-y}{\begin{vmatrix} 5 & 2 \\ -6 & -4 \end{vmatrix}} = \frac{z}{\begin{vmatrix} 5 & -6 \\ -6 & 4 \end{vmatrix}}$$

$$\frac{x}{\cancel{16}_{2}} = \frac{\cancel{-}y}{\cancel{-8}} = \frac{z}{\cancel{-16}_{2}}$$

$$\Rightarrow \frac{x}{2} = \frac{y}{1} = \frac{z}{-2}$$

$$X_2 = (2, 1, -2)$$

24
16
18
12
14

Case 3:- $\rightarrow \lambda = 15$

$$-7x - 6y + 2z = 0$$
$$-6x - 8y - 4z = 0$$
$$2x - 4y - 12z = 0$$

$$\frac{-x}{\begin{vmatrix}-6 & +2\\ -8 & -4\end{vmatrix}} = \frac{-y}{\begin{vmatrix}-7 & 2\\ -6 & -4\end{vmatrix}} = \frac{z}{\begin{vmatrix}-7 & -6\\ -6 & -8\end{vmatrix}}$$

$$\frac{-x}{\cancel{40}\ 2} = \frac{-y}{\cancel{40}\ 2} = \frac{z}{\cancel{20}\ 1}$$

$$x = -2, \quad y = -2, \quad z = 1$$

$$X_3 = (-2, -2, 1)$$

(2) Find the Eigen value & the corresponding Eigen vector for the matrix

$$A = \begin{bmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{bmatrix}$$

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 7-\lambda & -2 & 0 \\ -2 & 6-\lambda & -2 \\ 0 & -2 & 5-\lambda \end{vmatrix} = 0$$

$$(7-\lambda)[(6-\lambda)(5-\lambda)-4] + 2(-2(5-\lambda)+0) + 0 = 0$$
$$(7-\lambda)[30 - 11\lambda + \lambda^2 - 4] + 2[-10 + 2\lambda] = 0$$

$1-18+99-162$

$\overset{2}{26}\times 4$ 26×4 26×2

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$7\lambda^2-77\lambda$

$(7-\lambda)(\lambda^2-11\lambda+26)-20+4\lambda=0$

$-\lambda^3+18\lambda^2-99\lambda+162=0$

$\lambda^3-18\lambda^2+99\lambda-162=0$ (Check $\lambda=1$ or 2 or 3)

$\lambda=3$ we get $\lambda=3 \Rightarrow 0$

3	1	-18	+99	-162
	0	1	-45	162
	1	-15	+54	0

$\lambda^2-15\lambda+54=0$

$\lambda=6,\ \lambda=9$

Case 1) $\rightarrow \lambda=3$

$(7-\lambda)x-2y=0$

$-2x+(6-\lambda)y-2z=0$

$-2y+(5-\lambda)z=0$

~~Case 2) $\lambda=$~~

$(7-3)x-2y+0z=0$

$-2x+(6-3)y-2z=0$

$+0x-2y+2z=0$

$4x-2y+0z=0$

$-2x+3y-2z=0$

$0x-2y+2z=0$

$$\frac{x}{\begin{vmatrix}-2 & 0\\ 3 & -2\end{vmatrix}}=\frac{-y}{\begin{vmatrix}4 & 0\\ -2 & -2\end{vmatrix}}=\frac{z}{\begin{vmatrix}4 & -2\\ -2 & 3\end{vmatrix}}$$

-8

$$=\frac{x}{\cancel{4}}=\frac{+y}{\cancel{+8}\,2}=\frac{z}{\cancel{8}\,2}\Rightarrow \frac{x}{1}=\frac{y}{2}=\frac{z}{2}$$

Case 2) $\lambda = 6$

$$(7-\lambda)x - 2y = 0$$
$$-2x + (6-\lambda)y - 2z = 0$$
$$-2y + (5-\lambda)z = 0$$

$$1x - 2y = 0$$

$$-2x + 0 - 2z = 0$$
$$-2y - z = 0$$

$$\frac{x}{\begin{vmatrix} -2 & 0 \\ 0 & -2 \end{vmatrix}} = \frac{-y}{\begin{vmatrix} 1 & 0 \\ -2 & -2 \end{vmatrix}} = \cancel{\frac{z}{\begin{vmatrix} -2 & 0 \\ -2 & -1 \end{vmatrix}}} \quad \frac{z}{\begin{vmatrix} 1 & -2 \\ 2 & 0 \end{vmatrix}}$$

$$X_2 = \cancel{(4, -2, -4)} \quad (2, 1, -2)$$

Case 3) $\lambda = 9$

$$X_3 = (-2, -2, 1)$$

(B)

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

$$[A - \lambda I] = 0$$

$$\begin{bmatrix} -2-\lambda & 2 & -3 \\ 2 & 1-\lambda & -6 \\ -1 & -2 & 0-\lambda \end{bmatrix} = 0$$

$(1)^2 + (1)^2 + 17$

$1 + 18 + 45$

$27 +$

$-45)^3 - (45)^2 + 45 = 0$

$(45)^3 + (45)^2 - 7(45)$

$$(-2-\lambda)[(1-\lambda)-(0-\lambda) - 12(2-(0-\lambda)) + 3(-4-(1-\lambda)) = 0$$
$$(-2-\lambda)(1-\lambda^2) + 4 - 2\lambda - 2(3\lambda) - 4 = 0$$
$$-2\lambda + 2\lambda^2 + 24 - \lambda^2 + \lambda^3 + 12\lambda + 4\lambda + 2(-13)$$
$$-\lambda^3 + \lambda^2 + 17\lambda + 45 = 0$$
$$-\lambda^3 - \lambda^2 + 7\lambda + 45 = 0$$
$$\ldots + \lambda - 45 = 0$$

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$$\begin{bmatrix} -2-\lambda & 2 & -3 \\ 2 & 1-\lambda & -6 \\ -1 & -2 & 0-\lambda \end{bmatrix} = 0$$

$$(-2-\lambda)\left[(1-\lambda)(-\lambda)-12\right] - 2(-2\lambda-6) - 3(-4+(1-\lambda)) = 0$$

$$(-2-\lambda)\left[\lambda^2-\lambda-12\right] + 4(\lambda+3) - 3(-3-\lambda) = 0$$

$$(-2-\lambda)\left[(\lambda+3)(\lambda-4)\right] + 4(\lambda+3) + 3(\lambda+3) = 0$$

$$(\lambda+3)\left[(-2-\lambda)(\lambda-4) + 4 + 3\right] = 0$$

$$(\lambda+3)\left[-2\lambda+8-\lambda^2+4\lambda+7\right] = 0$$

$$(\lambda+3)\left[-\lambda^2+2\lambda+15\right] = 0$$

$$(\lambda+3)(\lambda^2-2\lambda-15) = 0$$

$$(\lambda+3)(\lambda+3)(\lambda-5) = 0$$

$$\lambda = -3, \ \lambda = -3, \ \lambda = 5$$

$-2+3 \quad 2 \quad -3$
$2 \quad 1+3 \quad -6$
$-1 \quad -2 \quad 3$

$-2+(3) \quad 2 \quad -3$
2

~~Case 1, $\lambda = -3$~~

$$(2-\lambda)x + 2y - 3 = 0$$

$$2x + (1-\lambda)y - 6z = 0$$

$$-1x - 2y - \lambda z = 0$$

Case 1 $\rightarrow \lambda = -3$

$$x + 2y - 3z = 0$$

$$2x + 4y - 6z = 0 \Rightarrow x + 2y - 3z = 0$$

$$x - 2y + 3z = 0 \Rightarrow x + 2y - 3z = 0$$

$$x = 3z - 2y$$

$1, 2, -1$ Let $y = k_1$, $z = k_2$ $\quad X_1 = (3k_2 - 2k_1, k_1, k_2)$

$x = 3k_2 - 2k_1$ $\quad X_2 = (3k_2 - 2k_1, k_1, k_2)$

1 $A = \begin{bmatrix} 6 & -2 & +2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

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Modal Matrix

Diagonalization $D = P^{-1}AP \Rightarrow A = PDP^{-1}$

$P^{-1}BP \qquad A^n = PD^nP^{-1}$

~~Prob~~

1) Reduce the matrix $A = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$ to the diagonal form and hence find A^4.

$A = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$

$|A - \lambda I| = 0$

$\begin{vmatrix} -1-\lambda & 3 \\ -2 & 4-\lambda \end{vmatrix} = 0$

$|A - \lambda I| = 0$

$\begin{vmatrix} -1-\lambda & 3 \\ -2 & 4-\lambda \end{vmatrix} = 0$

$(-1-\lambda)(4-\lambda) + 6 = 0$

$-4 - 4\lambda + \lambda + \lambda^2 + 6 = 0$

$\lambda^2 - 3\lambda + 2 = 0$

$\lambda = 1, 2$

$-1-2$

$-3x+$

$4-2=2$

$-3x+2y$

$\left.\begin{aligned} (-1-\lambda)x + 3y = 0 \\ -2x + (4-\lambda)y = 0 \end{aligned}\right\} \rightarrow (1)$

Case 1) $\rightarrow \lambda = 1$

$-2x + 3y = 0$

$-2x + 3y = 0$

$\cancel{+}2x = \cancel{+}3y$

$\frac{x}{3} = \frac{y}{2}$

$-2x = -3y$

$\frac{x}{3} = \frac{y}{2}$

$$X_1 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

Case 2) $\rightarrow \lambda = 2$

$$-3x + 3y = 0$$
$$2x + 2y = 0$$

$$-3x = -3y$$
$$\frac{x}{\cancel{3}_1} = \frac{y}{\cancel{3}_1}$$
$$x = 1, \quad y = 1$$
$$X_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

<u>Modal matrix</u>

$$P = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$$

$$D = P^{-1}AP$$

$$P^{-1} = \frac{1}{|P|} \text{Adj } P$$

$$|P| = 1$$

$$\text{Adj } P = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$$

$$P^{-1} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$$

$$D = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

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$D = P^{-1}AP$
$P^{-1}AP = \text{diag}(1,2)$
$A^4 = PD^4 P^{-1}$

$3-0 \quad 0-16$
$2-0 \quad 0+32$

$3 \quad +16$
$2 \quad +16$

$\begin{bmatrix} 3 & 16 \\ 2 & 16 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$

$\begin{bmatrix} 3-32 & -3+48 \\ 2-32 & -2+48 \end{bmatrix}$

$\begin{bmatrix} -29 & 45 \\ -30 & 46 \end{bmatrix}$

$$D^4 = \begin{bmatrix} 1^4 & 0 \\ 0 & 2^4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 16 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 16 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} -29 & 45 \\ -30 & 46 \end{bmatrix}$$

(2) Diagonalise the mat

$$\begin{bmatrix} -19 & 7 \\ -42 & 16 \end{bmatrix}$$

$|A - \lambda I| = 0$

42×7
294

$$\begin{vmatrix} -19-\lambda & 7 \\ -42 & 16-\lambda \end{vmatrix} = 0$$

$(-19-\lambda)(16-\lambda) + 294 = 0$
$-304 - 19\lambda + 16\lambda - \lambda^2 + 294$
$+10 + 3\lambda + \lambda^2$
$\lambda^2 + 3\lambda - 10 = 0$
$\lambda = (2, -5)$

Case I $\rightarrow \lambda = 2$
$(-19-\lambda)x + 7y = 0$
$(-19-2)x + 7y = 0$
$21x + 7y = 0$

$$\frac{x}{\cancel{21}_3} = \frac{y}{\cancel{7}} = 0$$

$$x_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

Case 2) $\lambda = -5$

$$(-19+5)x + 7y = 0$$

$$(-14)x + 7y = 0$$

$$\frac{x}{-\cancel{14}_2} = \frac{y}{\cancel{7}_1}$$

$$x_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

Modal Matrix

$$P = \begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix}$$

$$P^{-1} = \frac{1}{|P|} \text{Adj } P$$

$$= (-1) \begin{bmatrix} 2 & -1 \\ -3 & 1 \end{bmatrix}$$

$$P^{-1} = \begin{bmatrix} -2 & 1 \\ 3 & -1 \end{bmatrix}$$

$$D = P^{-1} A P$$

$$= \begin{bmatrix} -2 & 1 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} -19 & 7 \\ -42 & 16 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 38-42 & -14+16 \\ -57+42 & 21-16 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & +2 \\ -15 & 5 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix}$$

$$\begin{bmatrix} -4+6 & -4+4 \\ -15+15 & -15+10 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & -5 \end{bmatrix}$$

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Reduce the matrix

$$A = \begin{bmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{bmatrix}$$ into a diagonal matrix also find A^5

$|A - \lambda I| = 0$

$$\begin{vmatrix} 11-\lambda & -4 & -7 \\ 7 & -2-\lambda & -5 \\ 10 & -4 & -6-\lambda \end{vmatrix} = 0$$

$$(11-\lambda)\left[(-2-\lambda)(-6-\lambda)-20\right] + 4\left[7(-6-\lambda)+50\right] - 7\left[-28-10(-2-\lambda)\right] = 0$$

$$(11-\lambda)\left[12+8\lambda+\lambda^2-20\right] + 4\left[-42-7\lambda+50\right] + 7\left[-28+20+10\lambda\right]$$

$$(11-\lambda)(\lambda^2+8\lambda-8) + 4\left[-7\lambda+8\right] - 7\left[10\lambda-8\right] = 0$$

$$11\lambda^2 + 88\lambda - 88 - \lambda^3 - 8\lambda^2 + 8\lambda - 28\lambda + 32 - 70\lambda + 56 = 0$$

$$-\lambda^3 + 3\lambda^2 - 2\lambda = 0$$

$$-\lambda\left[\lambda^2 + 3\lambda - 2\right] = 0$$

$$\lambda = 0, 1, 2$$

$$\left.\begin{aligned} (11-\lambda)x - 4y - 7z &= 0 \\ 7x + (-2-\lambda)y - 5z &= 0 \\ 10x - 4y + (-6-\lambda)z &= 0 \end{aligned}\right\} \rightarrow (1)$$

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Case 1) $\rightarrow \lambda = 0 \Rightarrow$
$$11x - 4y - 7z = 0$$
$$7x - 2y - 5z = 0$$
$$10x - 4y - 6z = 0$$

$$\frac{x}{\begin{vmatrix} -4 & -7 \\ -2 & -5 \end{vmatrix}} = \frac{-y}{\begin{vmatrix} 11 & -7 \\ 7 & -5 \end{vmatrix}} = \frac{z}{\begin{vmatrix} 11 & -4 \\ 7 & -2 \end{vmatrix}}$$

$$\frac{x}{\cancel{6}_1} = \frac{\cancel{-}y}{\cancel{-6}_1} = \frac{z}{\cancel{6}_1}$$

$$X_1 = (1,1,1) \quad (\text{or}) \quad X_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Case 2) $\rightarrow \lambda = 1 \Rightarrow$
$$10x - 4y - 7z = 0$$
$$7x - 3y - 5z = 0$$
$$10x - \cancel{4}y - 7z = 0$$

-7-0

-4-1

-6-1

$$\frac{x}{\begin{vmatrix} -4 & -7 \\ -3 & -5 \end{vmatrix}} = \frac{-y}{\begin{vmatrix} 10 & -7 \\ 7 & -5 \end{vmatrix}} = \frac{z}{\begin{vmatrix} 10 & -4 \\ 7 & -3 \end{vmatrix}}$$

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-30

$$\frac{x}{1} = \frac{-y}{1} = \frac{z}{2} \Rightarrow X_2 = (-1,+1,-2) \Rightarrow (1,-1,2)$$

Case 3) $\rightarrow \lambda = 2 \Rightarrow$
$$9x - 4y - 7z = 0$$
$$7x - 4y - 5z = 0$$
$$10x - 4y - 8z = 0$$

2-1

6

$$\frac{x}{\begin{vmatrix} -4 & -7 \\ -4 & -5 \end{vmatrix}} = \frac{-y}{\begin{vmatrix} 9 & -7 \\ 7 & -5 \end{vmatrix}} = \frac{z}{\begin{vmatrix} 9 & -4 \\ 7 & -4 \end{vmatrix}}$$

$$\frac{x}{8} = \frac{\cancel{-}y}{\cancel{-}4} = \frac{z}{8} \Rightarrow \frac{x}{2} = \frac{y}{1} = \frac{z}{2}$$

$(2,1,2)$ //

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$$P = \begin{bmatrix} 1 & 1 & 2 \\ 1 & -1 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

$\Rightarrow 1(-2-2) - 1(2-1) + 2(2+1)$

$= 1(-4) - 1(1) + 2(3)$

$= -4 - 1 + 6$

$= \underline{1}$

$D = P^{-1}AP$

$P^{-1} = \dfrac{1}{|P|} \text{Adj } P$

$1(-2-2)$

-4

$|P| = 1$

$$\text{Adj } P = \begin{bmatrix} -4 & 2 & 3 \\ -1 & 0 & 1 \\ 3 & -1 & -2 \end{bmatrix}$$

$$P^{-1}AP = \begin{bmatrix} -4 & 2 & 3 \\ -1 & 0 & 1 \\ 3 & -1 & -2 \end{bmatrix} \begin{bmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{bmatrix} \begin{bmatrix} 1 & 1 & 2 \\ 1 & -1 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 \\ -1 & 0 & 1 \\ 6 & -2 & -4 \end{bmatrix} \begin{bmatrix} 1 & 1 & 2 \\ 1 & -1 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$D^5 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 32 \end{bmatrix}$$

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Rayleigh's power method :-

Find the largest hygen value and the corresponding hygen vector of the matrix A by power method.

★ ①

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

$$X^{(0)} = (1,0,0)'$$

$$AX^{(0)} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} = 2 \begin{bmatrix} 1 \\ 0 \\ 0.5 \end{bmatrix} = \cancel{\lambda}^{(1)} X^{(1)}$$

$$AX^{(1)} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0.5 \end{bmatrix} = \begin{bmatrix} 2.5 \\ 0 \\ 2 \end{bmatrix} = 2.5 \begin{bmatrix} 1 \\ 0 \\ 0.8 \end{bmatrix} = \lambda^{(2)} X^{(2)}$$

$$AX^{(2)} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0.8 \end{bmatrix} = \begin{bmatrix} 2.8 \\ 0 \\ 2.6 \end{bmatrix} = 2.8 \begin{bmatrix} 1 \\ 0 \\ 0.928 \end{bmatrix} = \lambda^{(3)} X^{(3)}$$

$$AX^{(3)} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0.928 \end{bmatrix} = \begin{bmatrix} 2.928 \\ 0 \\ 2.856 \end{bmatrix} = 2.928 \begin{bmatrix} 1 \\ 0 \\ 0.975 \end{bmatrix}$$

$$AX^{(4)} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0.975 \end{bmatrix} = \begin{bmatrix} 2.975 \\ 0 \\ 2.95 \end{bmatrix} = 2.975 \begin{bmatrix} 1 \\ 0 \\ 0.991 \end{bmatrix}$$

$6+0.726+1.09 = 7.816$ $\qquad -2-1.089-0.545 = -3.634$

$2+0.363+1.635$

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$$Ax^{(5)} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0.99 \end{bmatrix} = \begin{bmatrix} 2.99 \\ 0 \\ 2.982 \end{bmatrix} = 2.99 \begin{bmatrix} 1 \\ 0 \\ 0.999 \end{bmatrix}$$

$$x = (1,0,1)'$$

$6+0.928+1.022 = 7.95$

$-2-1.392-0.511 = -3.903$

$2+0.464+1.533$
3.997

(2)

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} \rightarrow x = [1,1,1]'$$

$$\begin{bmatrix} 6-\cancel{2}+\cancel{2} \\ -2+3-1 \\ 2-1+3 \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \\ 4 \end{bmatrix}$$

$$Ax^{(0)} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \\ 4 \end{bmatrix} = 6 \begin{bmatrix} 1 \\ 0 \\ 0.66 \end{bmatrix} = \lambda^{(1)} x^{(1)}$$

6

$-4+2 \quad -2+3$

$1+3$

$$Ax^{(1)} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0.666 \end{bmatrix} = \begin{bmatrix} 7.332 \\ -2.666 \\ 3.998 \end{bmatrix} = 7.332 \begin{bmatrix} 1 \\ -0.363 \\ 0.545 \end{bmatrix} = \lambda^{(2)} x^{(2)}$$

$$Ax^{(2)} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ -0.363 \\ 0.545 \end{bmatrix} = \begin{bmatrix} 7.816 \\ -3.634 \\ 3.998 \end{bmatrix} = 7.816 \begin{bmatrix} 1 \\ -0.464 \\ 0.511 \end{bmatrix} = \lambda^{(3)} x^{(3)}$$

$$Ax^{(3)} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ -0.464 \\ 0.511 \end{bmatrix} = \begin{bmatrix} 7.95 \\ -3.903 \\ 3.997 \end{bmatrix} = 7.95 \begin{bmatrix} 1 \\ -0.490 \\ 0.502 \end{bmatrix} =$$

$$Ax^{(4)} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ -0.490 \\ 0.502 \end{bmatrix} = \begin{bmatrix} 7.984 \\ -3.972 \\ 3.996 \end{bmatrix} = 7.984 \begin{bmatrix} 1 \\ -0.497 \\ 0.500 \end{bmatrix}$$

(3)

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -4 \\ -2 & 1 & 5 \end{bmatrix} \rightarrow X = (1, 0.8, -0.8)$$

$2 + 2.4 + 3.2$

$$Ax^0 = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -4 \\ -2 & 1 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 0.8 \\ -0.8 \end{bmatrix} = \begin{bmatrix} 5.6 \\ 7.6 \\ -5.2 \end{bmatrix} = 7.6 \begin{bmatrix} 0.736 \\ 1 \\ -0.684 \end{bmatrix}$$

$$Ax^1 = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -4 \\ -2 & 1 & 5 \end{bmatrix} \begin{bmatrix} 0.736 \\ 1 \\ -0.684 \end{bmatrix} = \begin{bmatrix} 4.628 \\ 7.208 \\ -3.892 \end{bmatrix} = 7.208 \begin{bmatrix} 0.642 \\ 1 \\ -0.539 \end{bmatrix}$$

$$Ax^2 = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -4 \\ -2 & 1 & 5 \end{bmatrix} \begin{bmatrix} 0.642 \\ 1 \\ -0.539 \end{bmatrix} = \begin{bmatrix} 4.107 \\ 6.44 \\ -2.979 \end{bmatrix} = 6.44 \begin{bmatrix} 0.637 \\ 1 \\ -0.462 \end{bmatrix}$$

$$Ax^3 = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -4 \\ -2 & 1 & 5 \end{bmatrix} \begin{bmatrix} 0.637 \\ 1 \\ -0.462 \end{bmatrix} = \begin{bmatrix} 4.01 \\ 6.122 \\ -2.584 \end{bmatrix} = 6.122 \begin{bmatrix} 0.655 \\ 1 \\ -0.422 \end{bmatrix}$$

$2.548 + 1 + 0.462$

$1.274 + 3 + 1.848$

$-1.274 + 1 \; \cancel{+} -2.31$

$$Ax^4 = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -4 \\ & & \end{bmatrix}$$