Equilibrium Of forces

A body is said to be in equilibrium when its static position or its motion with uniform Velocity is not altered by the system of forces acting on it. The bodies are subjected to applied forces, self wt & reactions from the adjoining bodies in contact.

Any system of forces acting on a body is said to be in equilibrium when the resultant of the forces is zero & the algebraic sum of the forces should be equal to zero..

Conditions for equilibrium

If the body should be in equilibrium, it should satisfy the following conditions

$\Sigma H = 0$ (algebraic sum of horizontal of forces)

$\Sigma V = 0$ (— " — vertical forces)

$R = 0$ (Resultant)

$\Sigma M = 0$ (algebraic sum of moment of all the forces)

Types of forces acting on a body

→ Applied forces

→ Non applied forces → Self weight
→ Reactions

Applied forces :- Applied forces are the forces applied externally to a body. Each of the forces has got a point of contact with the body.

ex: If a person stands on a ladder, his wt is an applied force.

Self Weight :- Everybody is subjected to gravitational acceleration & hence has a self weight.

$$W = mg$$

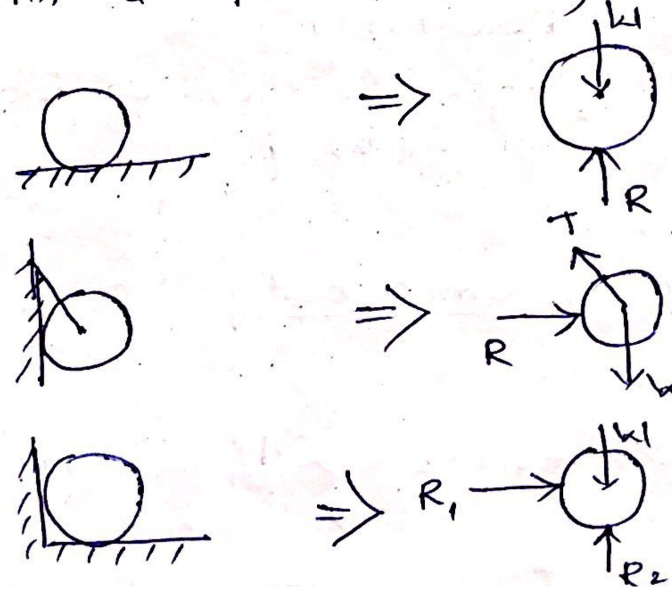
Self weight always acts vertically downwards, through centre of gravity.

Reactions :- Reactions are the self adjusting forces developed by other bodies which are in contact with the body ~~number~~ consideration.

Free body diagram :-

free body diagram of the body under consideration is freed from ~~the~~ the contact surfaces & with all the forces on it (including self wt, reactions & applied forces).

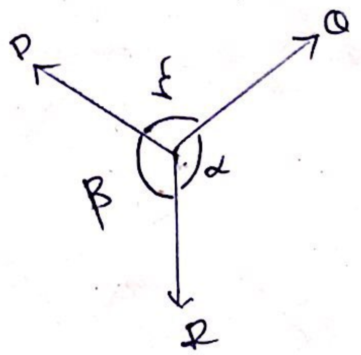
ex:

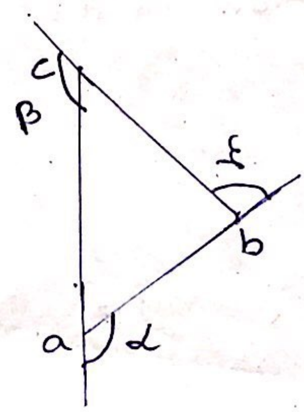


Note ; Strings , cable , wires are always subjected to tension only . ie the forces act outward
Spheres are subjected to inward reaction

Lami's theorem

"If three coplanar forces acting simultaneous @ a point be in equilibrium, then each force is proportional to the sine of the angle between the other two forces . .





Draw three forces P, Q, R one after the other in direction & magnitude starting from point a.
Since the body is in equilibrium , the resultant should be zero which means the last point of force is diagram should coincide with 'a'. Thus this results in a traingle of forces abc as shown in fig
now external angles @ a, b, & c are as shown in fig. In the Δ^{le} of forces abc,

$ab = Q$

$bc = P$

$ca = R$.

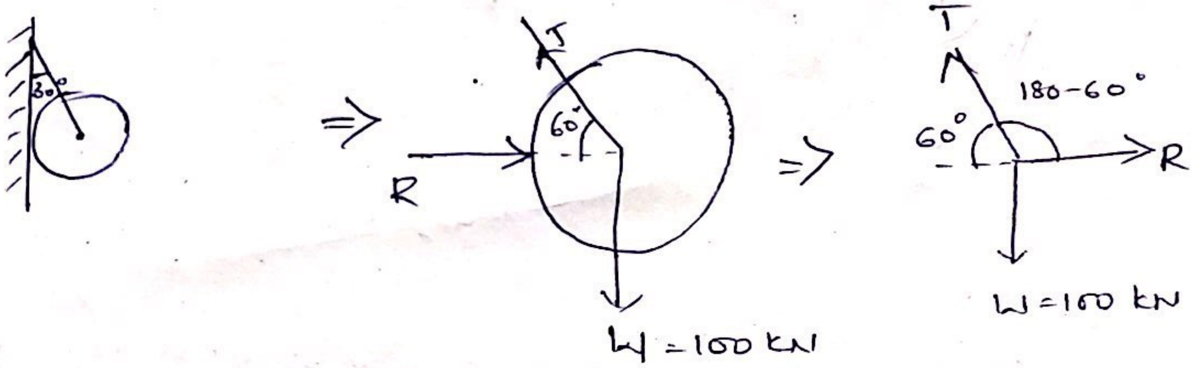
Apply Sine rule for the Δ^{le} abc,

$$\frac{ab}{\sin(180-\beta)} = \frac{bc}{\sin(180-\alpha)} = \frac{ca}{\sin(180-\gamma)}$$

$$\frac{Q}{\sin\beta} = \frac{P}{\sin\alpha} = \frac{R}{\sin\gamma}$$

Problems

1) A string is touching to the wall of weight 100kN. Find the tension in the wire which is supporting the sphere.



Apply lami's theorem

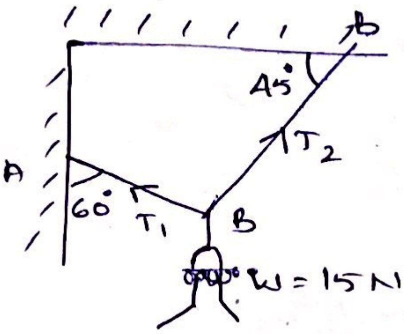
$$\frac{W}{\sin(180-60)} = \frac{T}{\sin 90^\circ} = \frac{R}{\sin(90+60)}$$

$$\frac{100}{\sin 120} = \frac{T}{\sin 90^\circ} = \frac{R}{\sin 150}$$

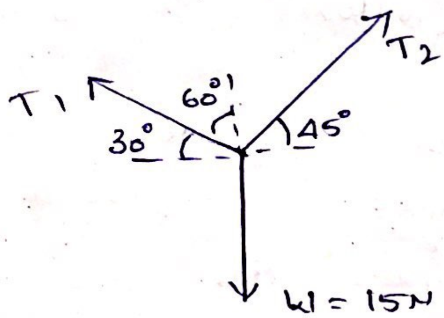
$$T = \frac{100}{\sin 120} \times \sin 90 = 115.47 \text{ kN}$$

$$R = \frac{100}{\sin 120} \times \sin 150 = 57.73 \text{ kN}$$

2) The figure shows, the lamp is fixed with string. Determine the tension in strings.



free body diagram



The system of 3 forces are in eqm

By Lami's theorem,

$$\frac{W}{\sin(60+45)} = \frac{T_2}{\sin(90+30)} = \frac{T_1}{\sin(90+45)}$$

$$\frac{15}{\sin 105} = \frac{T_2}{\sin 120} = \frac{T_1}{\sin 135}$$

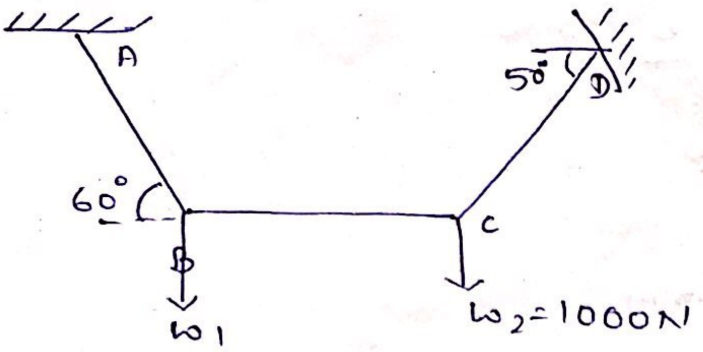
$$T_2 = \frac{15}{\sin 105} \times \sin 120$$

$$=$$

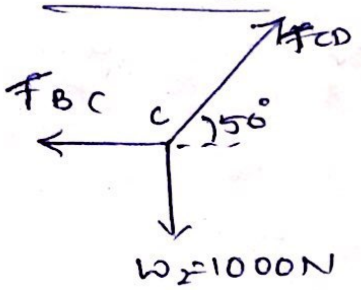
$$T_1 = \frac{15}{\sin 105} \times \sin 135.$$

$$=$$

3) find the forces in all the bars & the Load w_1 to keep the system in eq^m as shown.



f.B.D @ C



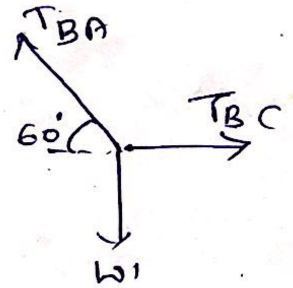
$$\frac{T_{CD}}{\sin 90°} = \frac{T_{BC}}{\sin(90+50)} = \frac{1000}{\sin(180-50)}$$

$$\frac{T_{CD}}{\sin 98} = \frac{T_{BC}}{\sin 140°} = \frac{1000}{\sin 130}$$

$$T_{CD} = \frac{1000}{\sin 130} \times \sin 90 = 1305.4N$$

$$T_{BC} = \frac{1000}{\sin 130} \times \sin 140 = 839.1N$$

f.B.D @ B



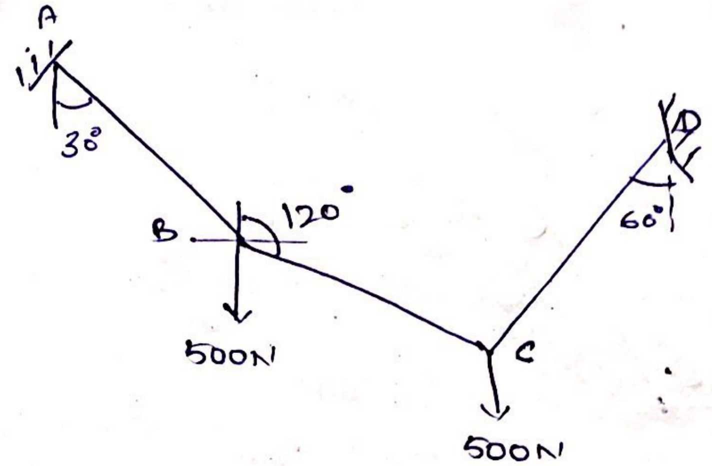
$$\frac{W_1}{\sin(180-60°)} = \frac{T_{BC}}{\sin(90+60)} = \frac{T_{BA}}{\sin 90°}$$

$$\frac{W_1}{\sin 120} = \frac{839.1}{\sin 150} = \frac{T_{BA}}{\sin 90°}$$

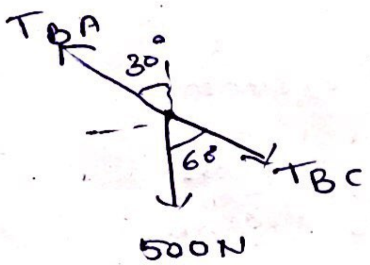
$$W_1 = \frac{839.1}{\sin 150} \times \sin 120 = 1453.36N$$

$$T_{BA} = \frac{839.1}{\sin 150} \times \sin 90° = 1678.2N$$

4) find the tension in the String shown in fig



Sol^n:- f.B.D @ point B.

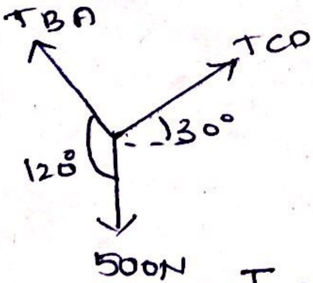


$$\frac{T_{AB}}{\sin 60} = \frac{T_{BC}}{\sin 150} = \frac{500}{\sin 150}$$

$$T_{AB} = \frac{500 \times \sin 60^\circ}{\sin 150} = 865N$$

$$T_{BC} = \frac{500 \times \sin 150}{\sin 150} = 500N$$

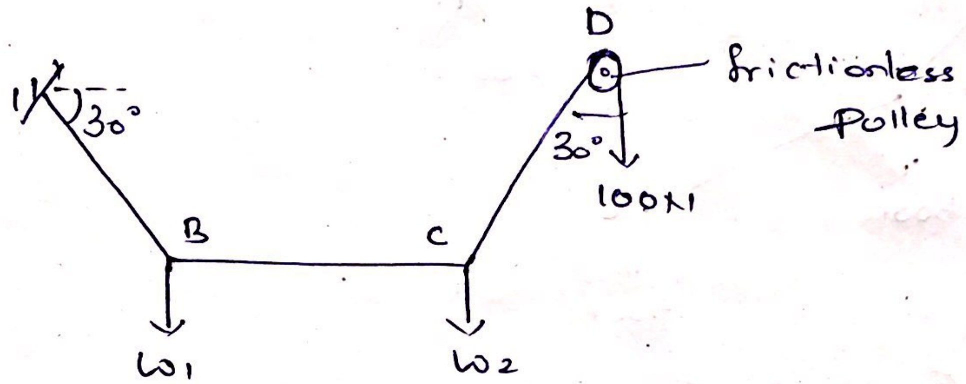
fB.D @ point C



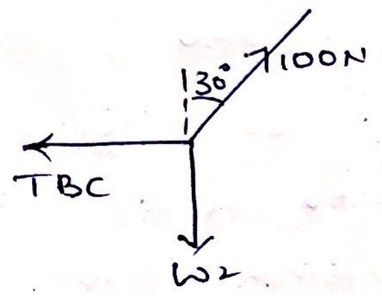
$$\frac{T_{BC}}{\sin 120^\circ} = \frac{T_{CD}}{\sin 120} = \frac{500}{\sin 120}$$

$$T_{CD} = \frac{T_{BC} \times \sin 120}{\sin 120} = 500$$

5) A light string ABCDE is fixed to weights W_1 & W_2 @ B & C a weight 100N @ free end E. find the tension in the string & weight W_1 & W_2.



Sol^n:- free body diagram @ C



By Lami's theorem

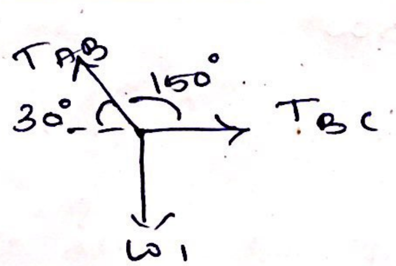
$$\frac{100}{\sin 90°} = \frac{T_{BC}}{\sin 150} = \frac{W_2}{\sin 120}$$

$$T_{BC} = \frac{100 \times \sin 150°}{\sin 90}$$

$$T_{BC} = 50N$$

$$W_2 = \frac{100}{\sin 90} \times \sin 150° = 86.6N$$

f.B.D @ B

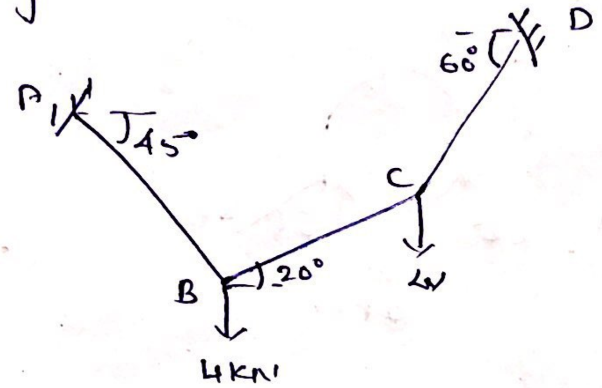


$$\frac{T_{AB}}{\sin 90°} = \frac{T_{BC}}{\sin 120°} = \frac{W_1}{\sin 150°}$$

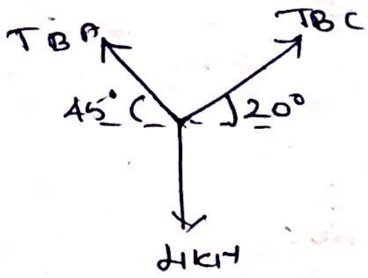
$$T_{AB} = \frac{50 \times \sin 90°}{\sin 120°} = 57.78N$$

$$W_1 = \frac{T_{BC}}{\sin 120°} \times \sin 150° = 28.86N.$$

6) A String is subjected to the forces 4KN & W as shown in fig. Determine the magnitude of W & the tension induced in various portions of the String.



FBD @ B



By Lami's theorem

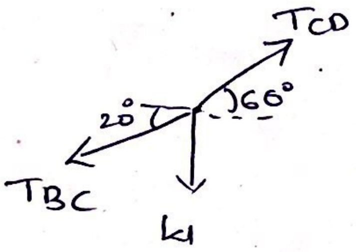
$$\frac{T_{AB}}{\sin(90+20)} = \frac{T_{BC}}{\sin(90+45)} = \frac{4}{\sin(180-45-20)}$$

$$\frac{T_{AB}}{\sin 110} = \frac{T_{BC}}{\sin 135} = \frac{4}{\sin 115}$$

$$T_{AB} = \frac{4}{\sin 115} \times \sin 110 = 4.15N$$

$$T_{BC} = \frac{4}{\sin 115} \times \sin 135 = 3.125 N$$

FBD @ C



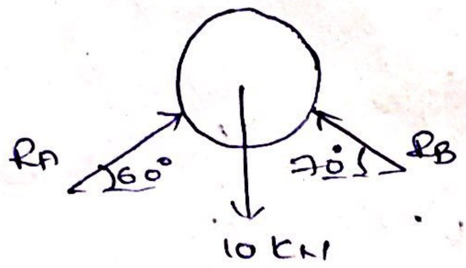
$$\frac{T_{BC}}{\sin(90+60)} = \frac{T_{CD}}{\sin(90-20)} = \frac{W}{\sin 140}$$

$$\frac{T_{BC}}{\sin 150} = \frac{T_{CD}}{\sin 70} = \frac{W}{\sin 140}$$

$$T_{CD} = \frac{3.125 \times \sin 70}{\sin 150} = 5.87KN$$

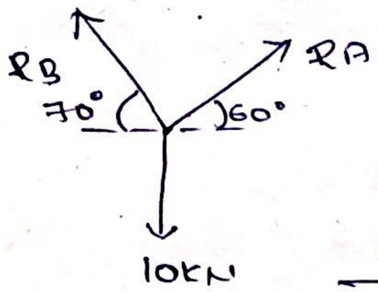
$$W = \frac{3.125 \times \sin 140}{\sin 150} = 4.02KN,$$

7) A Smooth circular cylinder of radius 15 m is placed in a notch as shown. find the reactions @ Contact Surface



Soln :-

f.B.D



$$\frac{R_A}{\sin(10+70)} = \frac{10}{\sin(180-60-70)} = \frac{R_B}{\sin(60+}$$

$$\frac{R_A}{\sin 160} = \frac{R_B}{\sin 150} = \frac{10}{\sin 50°}$$

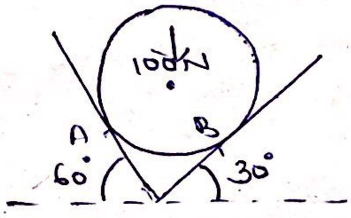
$$R_A = \frac{10 \times \sin 160°}{\sin 50°} =$$

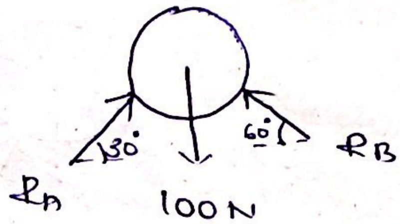
$$R_A = 4.46 \text{ kN}$$

$$R_B = \frac{10 \times \sin 150}{\sin 50}$$

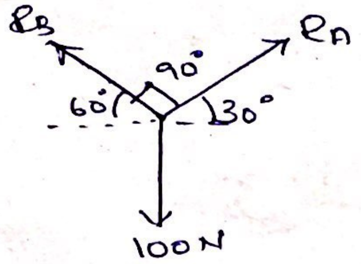
$$= 6.54 \text{ kN}$$

8) A sphere weighing 100N is fitted in a right angled notch as shown. Determine the reactions @ A & B.





FBD



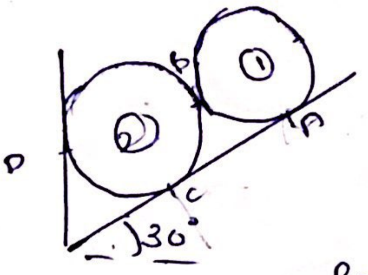
By Lami's theorem

$$\frac{R_B}{\sin(90+30)} = \frac{R_A}{\sin(90+60)} = \frac{100}{\sin 90^\circ}$$

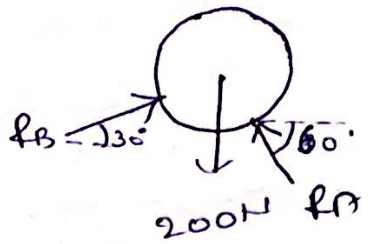
$$R_A = \frac{100}{\sin 90} \sin(90+60) = 49.98N$$

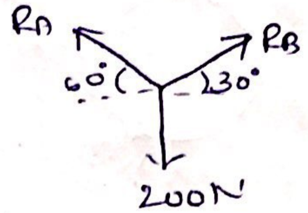
$$R_B = \frac{100}{\sin 90} \sin(90+30) = 86.62N$$

7) Two Identical rollers each weighing 200N are placed in a trough as shown in fig.



F.B.D for Sphere 1



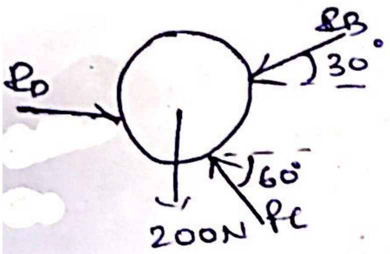


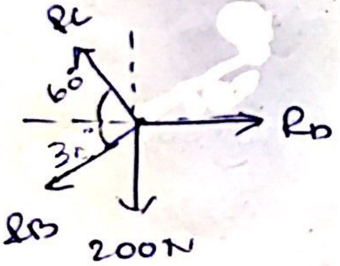
$$\frac{R_A}{\sin(90+30)} = \frac{R_B}{\sin(90+60)} = \frac{200}{\sin 90}$$

$$R_A = \frac{200}{\sin 90} \times \sin(90+30) = 17.3N$$

$$R_B = \frac{200}{\sin 90} \times \sin(90+60) = 100N$$

F.B.D for Sphere 2





The System is in eq^m

$\Sigma V = 0$

$-R_B \sin 30 + R_C \sin 60° - 200 = 0$

$-100 \sin 30° + R_C \sin 60° - 200 = 0 \rightarrow R_C \sin 60 = 250$ —(1)

$\Rightarrow R_C = 288.67N$

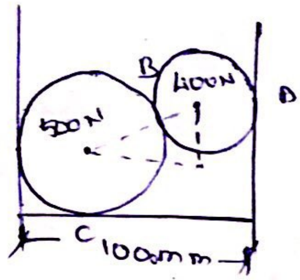
$\Sigma H = 0$

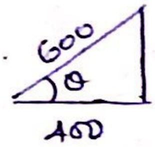
$R_D - R_B \cos 30° - R_C \cos 60° = 0$

$R_D - 100 \cos 30° - 288.67 \cos 60° = 0$

$R_D = 230.94\ N.$

10) Two spheres are placed in a canal as shown in fig. find the reactions @ all the point of contacts.

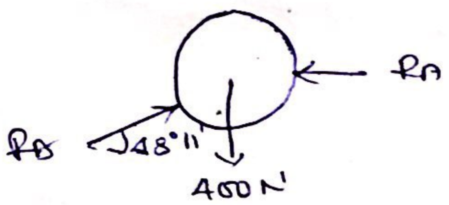


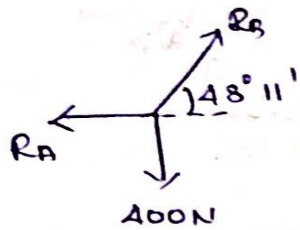


$$\cos\theta = \frac{400}{600}$$

$$\theta = 48°11'$$

FBD for sphere 1



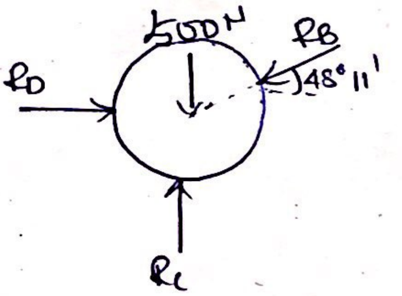


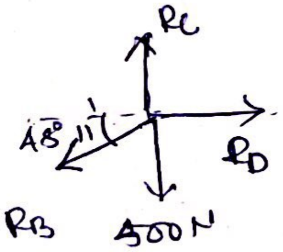
$$\frac{400}{\sin(180° - 48°11')} = \frac{R_B}{\sin 90°} = \frac{R_A}{\sin(90+48°11')}$$

$$R_A = \frac{400}{\sin 131°48'} \times \sin(90+48°11') = 357.77N$$

$|||^{ly}$ $R_B = 536.66 N$

FBD for sphere 2





$\Sigma H = 0$

$R_D - R_B \cos 48°11' = 0$

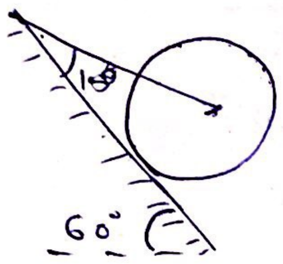
$R_D = 357.82 N$

$\Sigma V = 0$

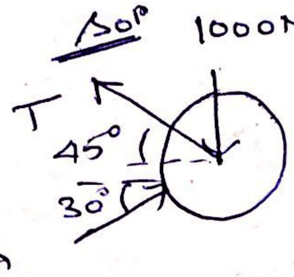
$R_C - 500 - R_B \sin 48°11' = 0$

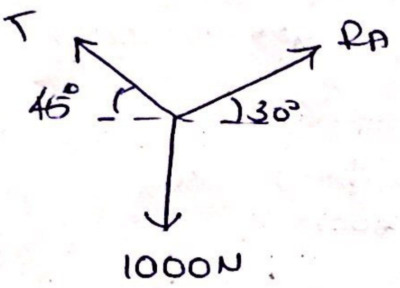
$R_C = 900 N.$

11) Determine the tension & reaction @ the contact surface for the cylinder of weight 1000N placed as shown.



Sol





$$\frac{R_A}{\sin(90°+45°)} = \frac{T}{\sin 120} = \frac{1000}{\sin 105}$$

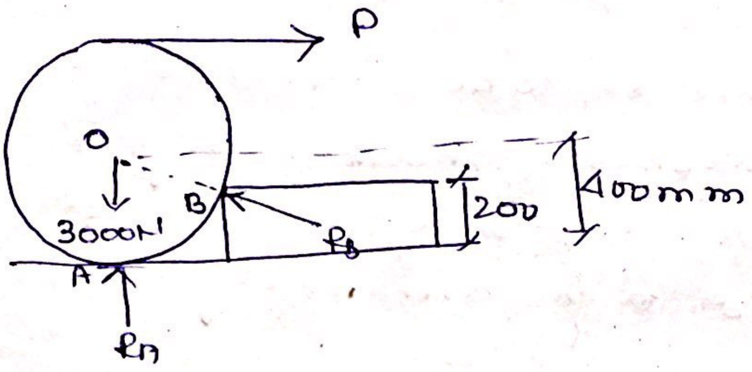
$$T = \frac{1000}{\sin 105°} \times \sin 120°$$

$$=$$

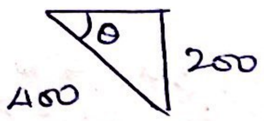
$$R_A = \frac{1000}{\sin 105} \times \sin 135°$$

$$=$$

12) A roller of radius 400mm weighing 3000N is to be pulled over a block of height 200mm as shown by a horizontal force P. Determine the magnitude of P, R_A & R_B.



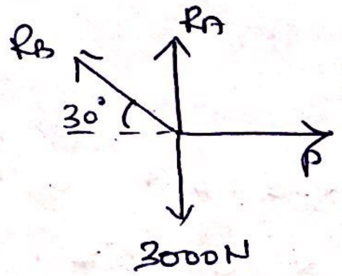
Soln



$$\sin\theta = \frac{200}{400}$$

$$\theta = 30^\circ$$

F.BD



$$\Sigma V = 0$$

$$R_A - 3000 + R_B \sin 30^\circ = 0$$

Since the roller is pulled, no contact surface exists @ A

$$\Rightarrow R_A = 0$$

$$0 - 3000 + R_B \sin 30^\circ = 0$$

$$\Rightarrow R_B = 6000\,N$$

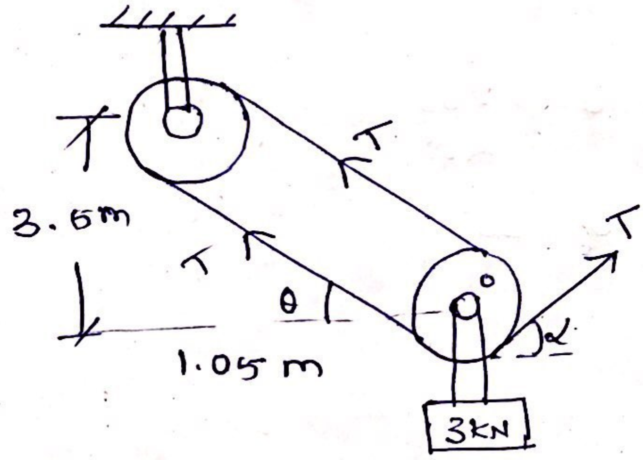
$\Sigma H = 0$

$P - R_B \cos 30° = 0$

$P - 6000 \cos 30° = 0$

$P = 5196.2\ N$

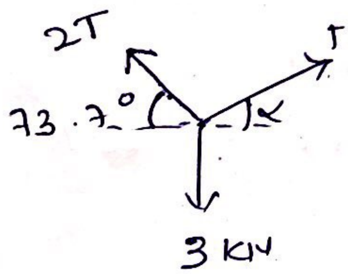
13) A 3KN crate is to be supported by the rope & pulley arrangement shown in fig. Determine the magnitude & direction of the force T, which should be exerted @ the free end of the rope.



$\tan\theta = \frac{3.6}{1.05}$

$\theta = 73.7°$

F.B.D @ O



$\Sigma H = 0$

$T\cos\alpha - 2T\cos 73.7 = 0$

$\Rightarrow \cos\alpha = 2\cos 73.7$

$\alpha = \cos^{-1}(2\cos 73.7) = \cos^{-1}(0.56)$

$\alpha = 55.9°$

$\Sigma V = 0$

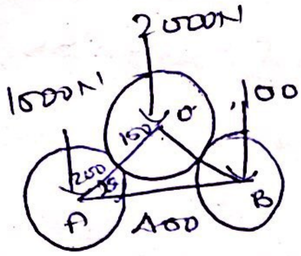
$T\sin\alpha + 2T\sin 73.7 = 0$

$T\sin 55.9 + 2T\sin 73.7 = 0$

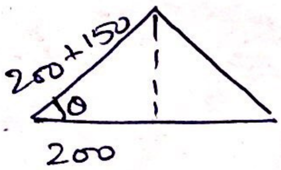
$\Rightarrow T = 1.09\ KN$

14) 2 smooth circular cylinder each of weight 1000 N & radius 200 mm are connected by the string AB of length 400 mm & rest upon the horizontal floor. Supporting above them in an another cylinder of weight 2000 N & radius 150 mm. Determine the tension in string & reaction developed @ contact surface.

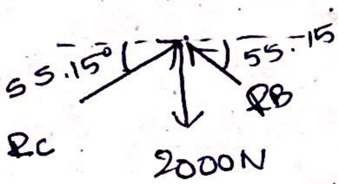
Soln :-



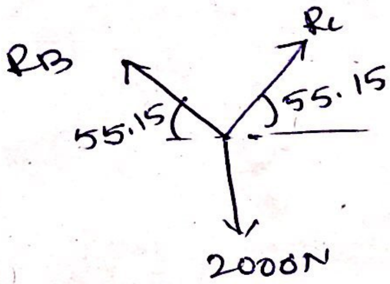
$\cos\theta = \frac{200}{350} \Rightarrow \theta = 55.15^\circ$



F.BD @ O.



$\rightarrow$

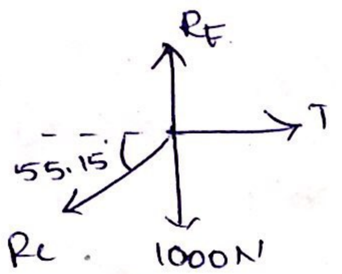


$$\frac{R_B}{\sin(90+55.15)} = \frac{R_C}{\sin(90+55.15)} = \frac{2000}{\sin 69.7}$$

$R_C = 1218.54$ N

$R_B = 1218.54$ N

f.B.D @ A



$\Sigma H = 0$

$T - R_C \cos 55.15 = 0$

$T = 696.31N$

$\Sigma V = 0$

$-1000 + R_E - R_C \sin 55.15 = 0$

$R_E = 1999.99N$

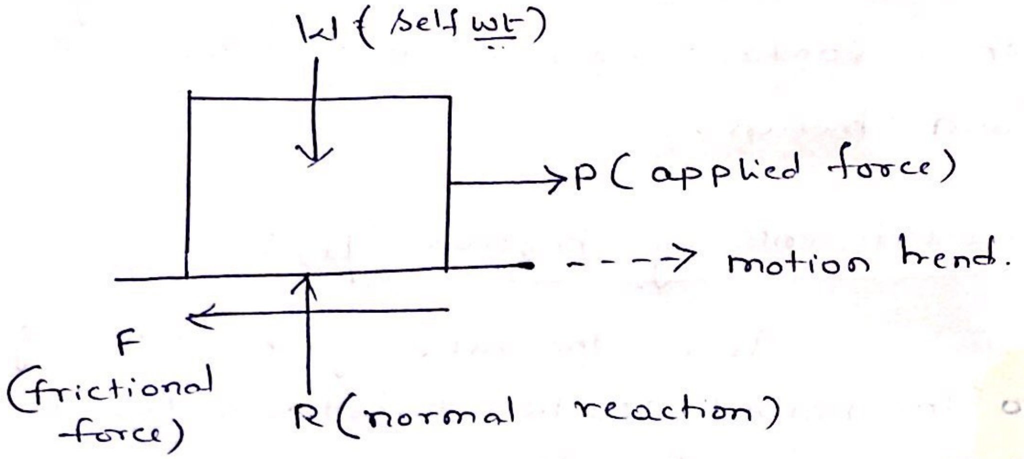
$\Rightarrow R_E = R_F = 1999.9N$

$R_C = R_D = 1218.54N$

$T = 696.31N$

Friction

- When one body tends to move in contact over other body a resistance to its movement is developed. This resistance to movement is called friction.



Types of friction

1) Static friction :- The friction acting along on a body which is @ rest is called static friction.

2) Limiting friction :- The friction acting on a body which is just on the point or verge of movement is called limiting friction.

3) Dynamic friction :- The friction acting on a body which is actually in motion is called dynamic friction. Dynamic friction is found to be less than the limiting friction.

Dynamic friction can be further classified into two types

→ Sliding friction :- Friction experienced by a body when it slides over the other body.

→ Rolling friction :- friction experienced by a body when it rolls over another body.

4) <u>Dry friction</u> :— The friction acting on a body when contact surfaces are dry & there is a tendency of relative motion is called dry friction. It is also called coulomb friction.

5) <u>Fluid friction</u> :— The friction acting on the body when contact surfaces are lubricated is called fluid friction.

<u>Co-efficient of friction (μ)</u> :—

It is the ratio of the limiting friction F to the normal reaction R between two surfaces. This is equal to the tangent of angle of friction

$$\mu = \frac{F}{R}$$

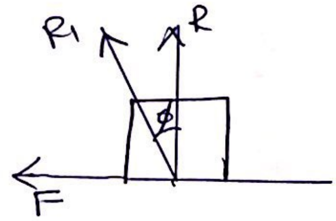
<u>Laws of friction</u> :—

1) The frictional force always acts in a direction opposite to that in which the body tends to move.
2) Till the limiting value is reached, the magnitude of frictional force is exactly equal to the tangential force which tends to move the body.
3) The magnitude of the limiting friction bears a constant ratio to the normal reaction between the two contacting surfaces.
4) The force of friction depends on the roughness of the surfaces.
5) The force of friction is independent of the area of contact between the two surfaces.

Angle of friction :-

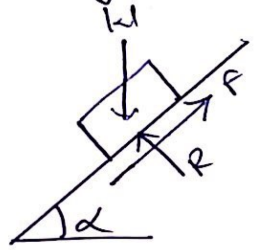
The angle which the resultant R_1 due to normal reaction & frictional force F makes with the normal to the surface is called the angle of friction.

$\tan\phi = \frac{F}{R}$



Angle of repose :-

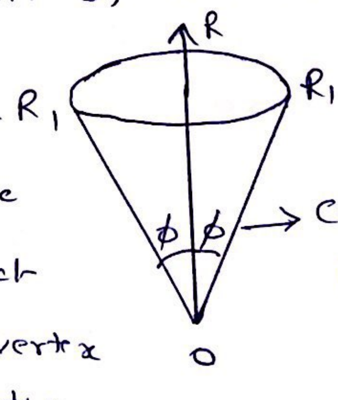
If a body is placed on an inclined plane, then the angle @ which the body is just on the point or verge of sliding down is called angle of Repose.



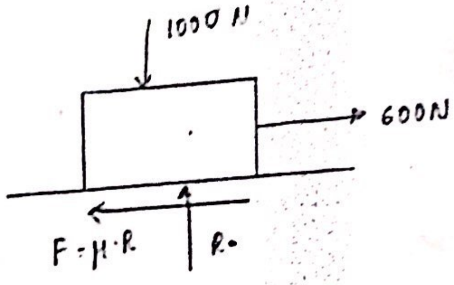
Cone of friction :-

Whenever a body is in contact with other tend to move, then the normal reaction OR & friction is developed as shown. These 2 forces can be replac by a resultant OR_1, when this resultant OR_1 making angle is revolved around point O, will form a right circular cone.

The cone having the point of contact as the vertex O, the normal OR @ the point of contact as its axis & ϕ as the semivertex angle is called the cone of friction.



1) A body of weight 1000N is placed on a rough horizontal plane. Determine the co-efficient of friction due to a force of 600N in horizontal direction just causes the body to slide.



Weight = 1000N

Pull = 600N.

μ = ?

Let R be the normal reaction and F is the frictional force.

$$\mu = \frac{F}{R}.$$

$$F = \mu \cdot R.$$

Resolving the forces vertically.

$$\Sigma V = 0$$

$$R - 1000 = 0$$

$$R = 1000\,N$$

Resolving the forces horizontally.

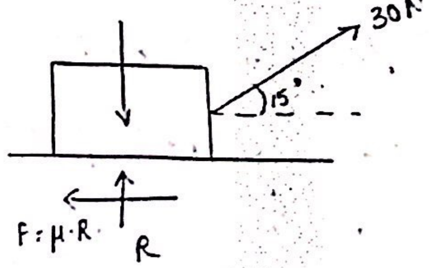
$$600 - F = 0$$

$$F = 600.$$

$$\mu = \frac{F}{R} = \frac{600}{1000}$$

$$\boxed{\mu = 0.6}$$

2) A body weighs 100N is placed on rough horizontal plane is pulled by a force of 30N inclined at 15° with horizontal. Find the co-efficient of friction.



W = 100N

P = 30N

θ = 15°

μ = ?

Resolving the forces vertically.

$$-100 + 30\sin 15° + R = 0$$

$$R = \underline{92.24\,N}$$

Resolving the forces horizontally

$\Sigma H = 0$

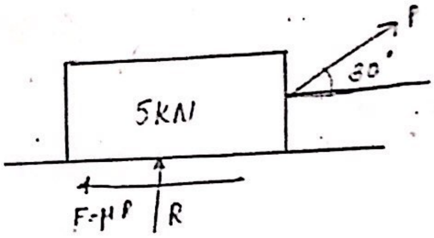
$$-F + 30\cos 15 = 0$$

$$F = 28.98\,N$$

$$\mu = \frac{F}{R} = \frac{28.98}{92.24} = \underline{0.314}$$

3) A block of weight 5KN rests on a horizontal surface and the co-efficient of friction b/n them is 0.4. Show that the magnitude of force required to pull is less than the magnitude of push.

(a) When pull is applied.



Resolving forces horizontally.

$\Sigma H = 0$,

$$P_2\cos 30^\circ - 0.4 R_2 = 0 \quad \text{------ (1)}$$

Resolving forces vertically.

$\Sigma V = 0$.

$$-5 + P_2\sin 30^\circ + R_2 = 0$$

$$R_2 = 5 - P_2\sin 30^\circ \quad \text{------ (2)}$$

substitute (2) in (1)

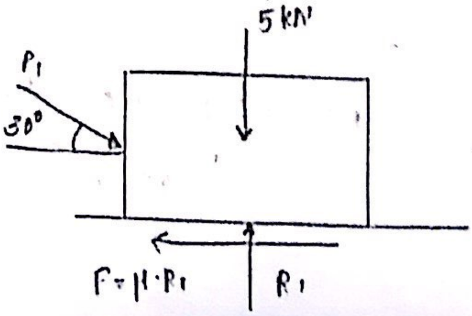
$$P\cos 30^\circ - 0.4(5 - P_2\sin 30^\circ) = 0$$

$$P_2\cos 30^\circ - 2 + 0.4\,P_2\sin 30^\circ = 0.$$

$$\boxed{P_2 = 1.876\,kN}$$

b) When push is applied.

Resolving the forces horizontally.



$\Sigma H = 0$.

$$P_1\cos 30^\circ - 0.4 R_1 = 0 \quad \text{------ (3)}$$

Resolving the forces vertically.
$\Sigma V = 0$

$R_1 - 5 - P_1 \sin 30° = 0$

$R_1 = 5 + P_1 \sin 30°$ ——— (4)

substituting (4) in (3)

$P_1 \cos 30° - 0.4(5 + P_1 \sin 30) = 0$

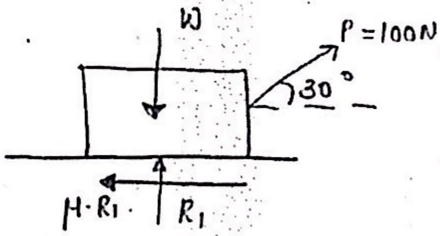
$\boxed{P_1 = 3.12 kN}$

from P_1 and P_2 it is clear that magnitude of force required to pull is less than the magnitude of push.

4) A body resting on a horizontal plane required a pull of 100N inclined at 30° to horizontal just to move it. It was found that a push of 110 N inclined at 20° to the plane just moved the body. Determine the weight of the body and co-efficient of friction.

Soln. When a pull of 100N inclined at 30° to plane.

Resolving the forces vertically $\Sigma V = 0$.



$-W + R_1 + 100 \sin 30 = 0$

$-W + R_1 + 50 = 0$ ——— (1)

Resolving the forces horizontally $\Sigma H = 0$.

$100 \cos 30° - \mu R_1 = 0$

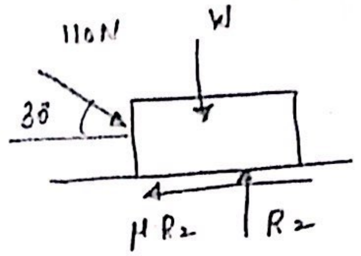
$R_1 = \frac{100 \cos 30}{\mu}$ ——— (2)

substituting (2) in (1)

$-W + \frac{86.6}{\mu} + 50 = 0$

$W - \frac{86.6}{\mu} = 50$

When a pull of 110N inclined at 20° with plane.



Resolving forces vertically $\Sigma V = 0$

$$-W - 110\sin 30 + R_2 = 0 \qquad \text{——— (3)}$$

Resolving forces horizontally $\Sigma H = 0$

$$110\cos 30 - \mu \cdot R_2 = 0$$

$$R_2 = \frac{103.37}{\mu} \qquad \text{——— (4)}$$

substituting (4) in (3)

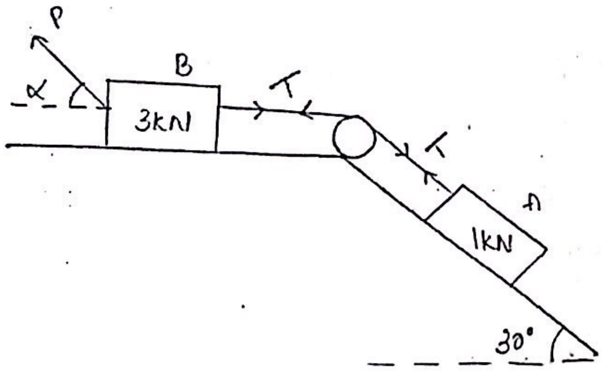
$$-W - 110\sin 30^\circ + \frac{103.37}{\mu} = 0$$

$$-W + \frac{103.37}{\mu} = 37.62$$

solving $\boxed{\mu = 0.19}$

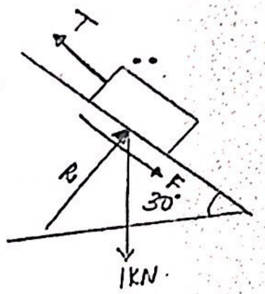
$\boxed{W = 506.38N}$

7) Find the least value of P required to cause the system of block shown in fig, to have impending motion to the left. Take co-efficient of friction for all contact surfaces as 0.2.



Soln :

Draw FBD for block A



Resolving the forces horizontally $\Sigma H = 0$.

$$-T + 0.2R_1 + 1\cos 60^\circ = 0$$

$$T - 0.2R_1 + \cos 60^\circ = 0$$

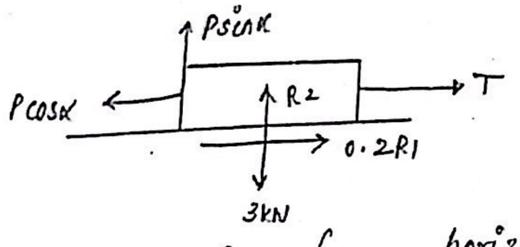
Resolving the forces vertically $\Sigma V = 0$.

$$R_1 - 1\sin 60^\circ = 0$$

$$\boxed{R_1 = 0.86\ KN}$$

$$\boxed{T = 0.672\ KN}$$

Draw FBD for Block B.



Resolving the forces horizontally

$$T - P\cos\alpha + 0.2R_1 = 0$$

$$0.672 - P\cos\alpha + 0.2R_1 = 0. \quad \text{——— (2)}$$

Resolving the forces vertically.

$$P\sin\alpha + R_2 - 3$$

$$R_2 = 3 - P\sin\alpha \quad \text{——— (3)}$$

By. (2) and (3)

$0.672 + 0.2(3 \cdot P\sin\alpha) - P\cos\alpha = 0$

$0.672 + 0.6 - 0.2P\sin\alpha - P\cos\alpha = 0$

for least value of P. $\frac{dP}{d\alpha} = 0$.

$0.2P\sin\alpha + P\cos\alpha = 1.272$ ---- (4)

Differentiating w.r to α.

$0.2P\cos\alpha - P\sin\alpha = 0$.

$0.2\cos\alpha - \sin\alpha = 0$

$\sin\alpha = 0.2\cos\alpha$.

$\frac{\sin\alpha}{\cos\alpha} = 0.2$

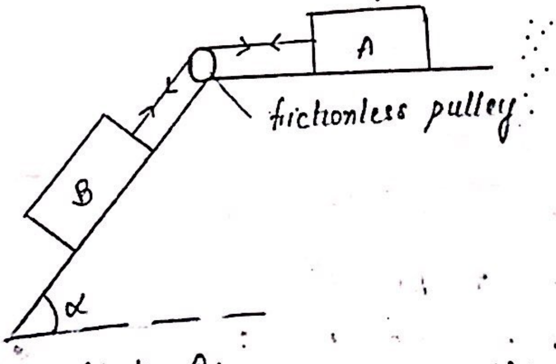
$\alpha = \tan^{-1}(0.2)$

$= 11°$.

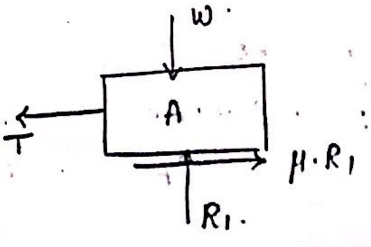
substituting α in (4)

$\boxed{P = 1.247 kN}$

8) Two blocks A and B weighing W_1 and W_2 are connected as shown in fig. If $W_1 = W_2$ and if μ is the co-efficient of friction for all contact surfaces. Find the angle of inclined plane α at which the motion of the system will impend.



Soln: Consider block A.



Resolving forces vertically.

$\Sigma V = 0$

$-W + R_1 = 0$

$W = R_1$ —— ①

Resolving forces horizontally $\Sigma H = 0$.

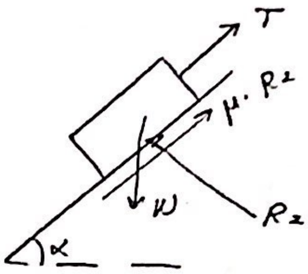
$$-T + \mu R_1 = 0$$

$$\boxed{T = \mu \cdot W} \quad \text{———— (2)}$$

Consider block B.

∴

Resolving forces horizontally $\Sigma H = 0$.



$$T + \mu \cdot R_2 - W \sin\alpha \quad \text{———— (3)}$$

Resolving forces vertically $\Sigma V = 0$

$$R_2 - W\cos\alpha = 0$$

$$R_2 = W\cos\alpha \quad \text{———— (4)}$$

from (2), (3) and (4)

$$\mu \cdot W + \mu \cdot [W\cos\alpha] - W\sin\alpha = 0.$$

$$\mu W[1 + \cos\alpha] = W\sin\alpha$$

$$\sin\alpha = \mu(1 + \cos\alpha)$$

$$\mu = \frac{\sin\alpha}{(1 + \cos\alpha)}$$

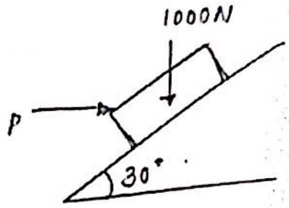
$$\mu = \frac{\cancel{2}\sin\alpha/2 \cos\alpha/2}{\cancel{2}\cos^2\alpha/2}.$$

$$\mu = \tan\alpha/2.$$

$$\frac{\alpha}{2} = \tan^{-1}(\mu)$$

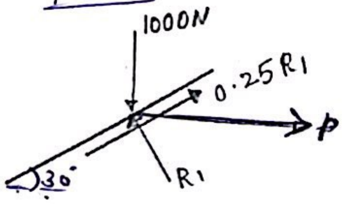
$$\alpha = 2\tan^{-1}(\mu).$$

6) A small block of weight 1000N is placed on a 30° incline with a co-efficient of friction at 0.25 as shown in fig. Determine the horizontal force to be applied for.
i) The impending motion down the plane and
ii) the impending motion up the plane.



Solⁿ.

i) The impending motion down the plane.



Resolving the forces horizontally $\Sigma H = 0$.

$$0.25R_1 + P\cos 30° - 1000\cos 60° = 0 \quad \text{——— (1)}$$

Resolving the forces vertically

$$R_1 - P\sin 30° - 1000\sin 60° = 0$$

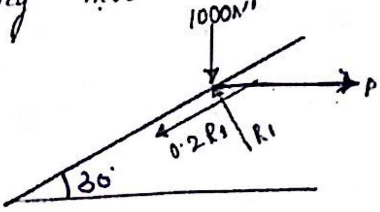
$$R_1 = P\sin 30° + 1000\sin 60° \quad \text{——— (2)}$$

substitute (2) in (1).

$$0.25[P\sin 30 + 1000\sin 60°] + P\cos 30° - 1000\cos 60 = 0$$

$$\boxed{P = 286.06\,N}$$

ii) The impending motion up the plane.



Resolving the forces vertically $\Sigma V = 0$.

$$R_1 - 1000\sin 60^\circ - P\sin 30 = 0.$$

$$R_1 = P\sin 30 + 1000\sin 60$$

Resolving the forces horizontally $\Sigma H = 0$.

$$P\cos 30 - 1000\cos 60^\circ - 0.2R_1 = 0$$

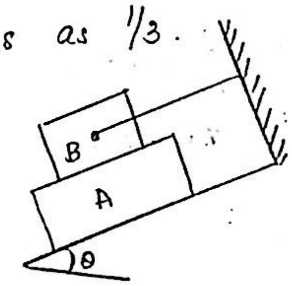
$$P\cos 30 - 1000\cos 60 - 0.2[P\sin 30 + 1000\sin 60^\circ] = 0.$$

$$\boxed{P = 966.91\,N}$$

7) What should be the value of 'θ' so that the motion of block A impends down the plane? The co-efficient of friction μ for all the surfaces as 1/3.

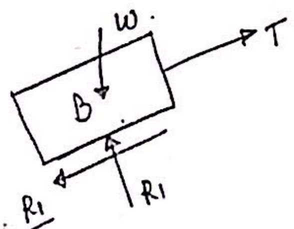
Mass of A = 40 kg.

Mass of B = 13.5 kg.



Soln:

Consider block B.



weight of block B = 13.5 × 9.81 = 132.43 N.

Resolving the forces vertically.

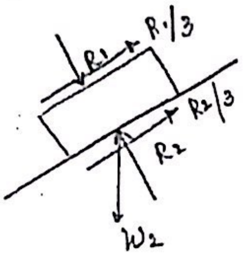
$$R_1 = 132.43\sin(90-\theta)$$

$$\boxed{R_1 = 132.43\cos\theta}$$

Resolving the forces horizontally $\Sigma H = 0$.

$$T - \frac{R_1}{3} - 132.43\cos(90-\theta) = 0$$

Consider block A.



weight of the block A

$= 40 \times 9.81 = 392.4\,N$.

Resolving the forces vertically

$$R_2 - R_1 - W_2 \sin(90-\theta) = 0$$

$$R_2 = W_2\cos\theta + R_1$$

$$R_2 = 392.4\cos\theta + 132.43\cos\theta$$

$$R_2 = 524.835\cos\theta.$$

Resolving the forces horizontally. $\Sigma H = 0$.

$$\frac{R_2}{3} + \frac{R_1}{3} = W_2\cos(90-\theta).$$

$$\frac{524.835\cos\theta}{3} + \frac{132.43\cos\theta}{3} = 392.4\sin\theta.$$

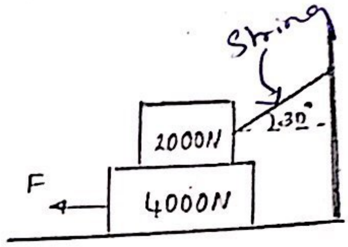
$$219.09\cos\theta = 392.4\sin\theta$$

$$\tan\theta = \frac{219.09}{392.4}$$

$$\theta = \tan^{-1}\left[\frac{219.09}{392.4}\right]$$

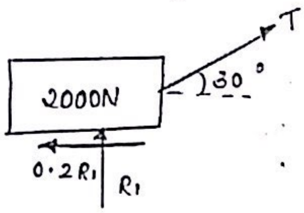
$$\theta = \underline{29^\circ 10'}$$

5) A block weighing 4000N is resting on horizontal surface supports another block of 2000N as shown in fig. Find the horizontal force F just to move the block to the left. Take μ for all contact surface as 0.2.



Soln:

Consider 2000N block.



Consider the FBD

Resolving the forces horizontally

$\Sigma H \Rightarrow T\cos 30° - 0.2R_1 = 0$

$$T = \frac{0.2R_1}{\cos 30°}$$

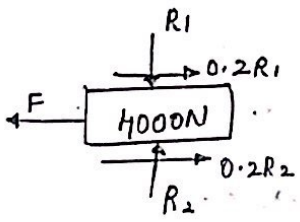
Resolving the forces vertically

$\Sigma V \Rightarrow R_1 - 2000N + T\sin 30° = 0.$

$R_1 - 20000 + 0.2\,R_1 \tan 30° = 0.$

$R_1 = 1792.97N.$

Consider 4000N block.



Resolving the forces horizontally.

$\Sigma H = 0.$

$0.2R_1 + 0.2R_2 - F = 0.$

$0.2(1792.9) + 0.2(5792.9) - F = 0.$

$\underline{F = 1517.19N}$

Resolving the forces vertically

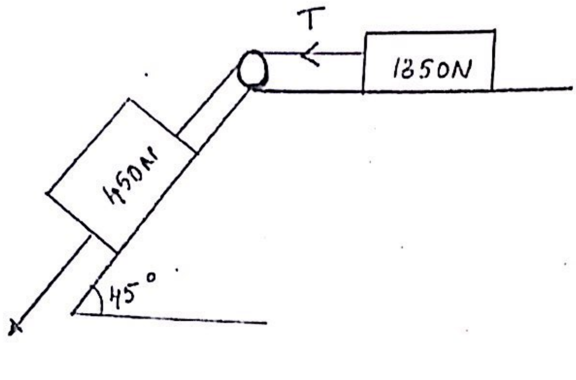
$\Sigma V = 0.$

$-R_1 + R_2 - 4000 = 0.$

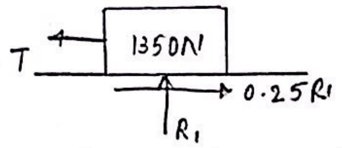
$-1792.9 + R_2 - 4000 = 0$

$R_2 = 5792.97N$

8) Determine the necessary force P acting parallel to the plane, as shown in fig in order to cause motion to impend. Take $\mu = 0.25$.



Soln: Consider 1350N block.



Resolving forces horizontally $\Sigma H = 0$.

$$-T + 0.25R_1 = 0$$

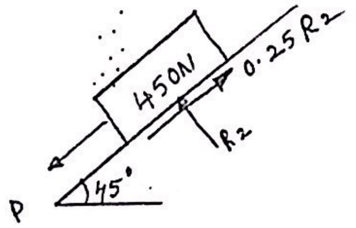
$$T = 0.25R_1 \quad \text{——} \quad (1)$$

Resolving forces vertically $\Sigma V = 0$.

$$R_1 = 1350 \quad \text{——} \quad (2)$$

substituting (2) in (1)

$$T = 0.25 \times 1350 = 337.5\,N.$$



Consider 450N block.

Resolving the forces vertically $\Sigma V = 0$.

$$-450\sin 45° + R_2 = 0$$

$$R_2 = \underline{318.198\ N}$$

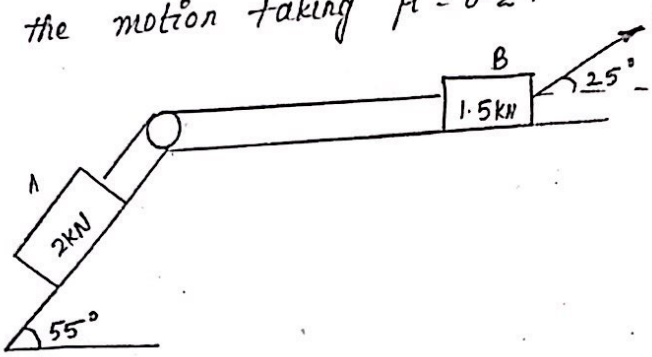
Resolving the forces horizontally $\Sigma H = 0$.

$0.25R_2 - P - 450\cos 45^\circ = 0$.

$0.25(318.192) - P - 450\cos 45^\circ = 0$

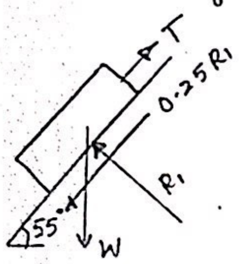
$P = 98.852\,N$.

9) Two blocks A and B weighing 2 kN and 1.5kN are connected by a wire passing over a smooth frictionless pulley as shown in fig. Determine the magnitude of force P required to impend the motion taking $\mu = 0.2$.



Soln:

Consider FBD of block A.



Resolving the forces horizontally $\Sigma H = 0$.

$T - 0.25R_1 - W\cos 35^\circ = 0$

$T = 2\cos 35^\circ + 0.25R_1$ —— (1)

Resolving the forces vertically $\Sigma V = 0$.

$R_1 = 2\sin 35^\circ$

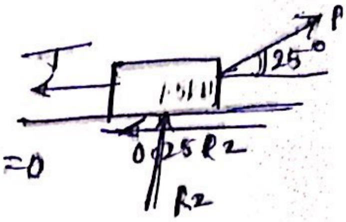
$R_1 = 1.147\,KN$ —— (2)

substitute 2 in (1)

$T = 2\cos 35^\circ + 0.25R_1$

$\boxed{T = 1.868\,KN.}$

Consider FBD of block B



Resolving the forces vertically $\Sigma V = 0$

$R_2 = 1.5kN - P\sin 25°$

Resolving the forces horizontally $\Sigma H = 0$.

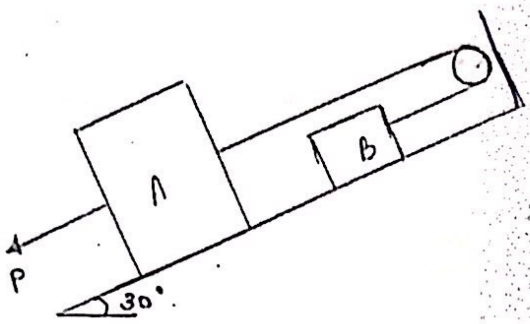
$P\cos 25° - T - 0.2R_2 = 0$

$P\cos 25° - T - 0.25[1.5 - P\sin 25°]$

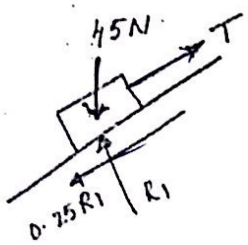
$\boxed{P = 2.188N}$

10) Determine the force P required to cause motion of blocks to impend. Take the weight of A as 90N and weight of B as 45N. Take the co-efficient of friction for all contact surfaces as 0.25. Consider the pulley being frictionless.

Soln :



Consider the block B



Resolving the forces vertically $\Sigma V = 0$.

$R_1 = 45\sin 60°$.

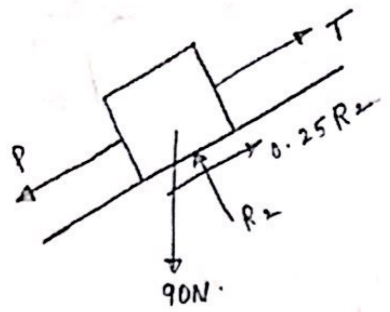
$R_1 = 38.97 N$

Resolving the forces horizontally $\Sigma H = 0$.

$T - 0.25R_1 - 45\cos 60°$

$T = 32.243N$.

Consider the block A.



Resolving the forces vertically $\Sigma V = 0$.

$$R_2 - 90\sin 60^\circ = 0$$

$$R_2 = 77.94N.$$

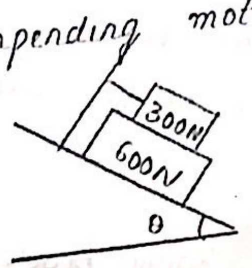
Resolving the forces horizontally $\Sigma H = 0$.

$$T - P - 90\cos 60^\circ + 0.25 R_2 = 0$$

$$32.243 - P - 90\cos 60^\circ + 0.25(77.94) = 0$$

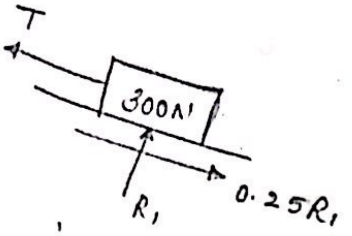
$$\boxed{P = 6.73N}$$

1) Determine the value of 'θ' for impending motion of the blocks. Take $\mu = 0.25$.



Soln : —

Consider block 300N



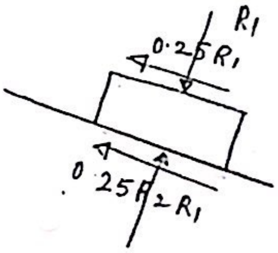
Resolving forces vertically $\Sigma V = 0$.

$$R_1 = 300\sin(90-\theta)$$
$$R_1 = 300\cos\theta$$

Resolving forces horizontally $\Sigma H = 0$.

$$0.25R_1 - T + 300\cos(90-\theta)$$
$$0.25[300\cos\theta] - T + 300\sin\theta$$
$$T = \_\_\_\_\_$$

Consider block 600N



Resolving forces vertically $\Sigma V = 0$.

$$R_2 - R_1 - 600\sin(90-\theta) = 0.$$
$$R_2 = 600\cos\theta + 300\cos\theta = 900\cos\theta.$$

Resolving forces horizontally $\Sigma H = 0$.

$$-0.25R_1 - 0.25R_2 + 600\sin\theta = 0$$

$$0.25(300\cos\theta) + 0.25(900\cos\theta) = 600\sin\theta$$

$$300\cos\theta = 600\sin\theta$$

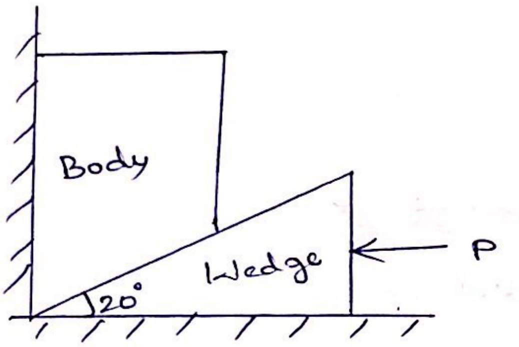
$$\frac{\sin\theta}{\cos\theta} = 1/2$$

$$\tan\theta = 1/2$$

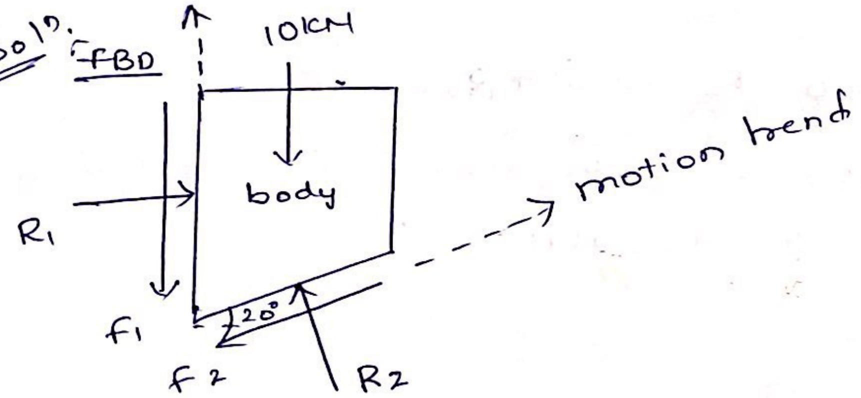
$$\boxed{\theta = 26^\circ 33'}$$

Problems on Wedge friction

1) A block weighing 10kN is to be raised by means of 20° Wedge as shown below. find the horizontal force which will just raise the block if co-efficient of friction for all surfaces of contact is 0.3.



Soln.: FBD



$f_2 = 0.3R_2$

$f_1 = 0.3R_1$

$\Sigma H = 0$

$$R_1 - R_2\sin 20^\circ - f_2\cos 20^\circ = 0$$

$$R_1 - 0.62R_2 = 0 \quad — \quad (1)$$

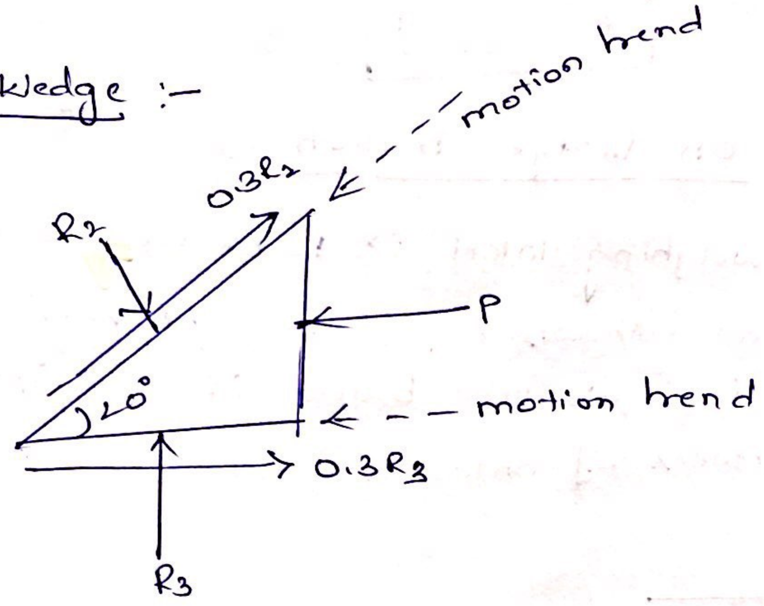
$\Sigma V = 0$

$-10 - 0.3R_1 + R_2\cos 20^\circ - 0.3R_2 \sin 20^\circ = 0$

$-0.3R_1 + 0.84R_2 = 10$ —— (2)

$R_2 = 15.29 \text{ kN}$.

FBD of Wedge :-



$\Sigma V = 0$

$R_3 - R_2\cos 20^\circ + 0.3R_2 \sin 20^\circ = 0$

$R_3 = 12.80 \text{ kN}$

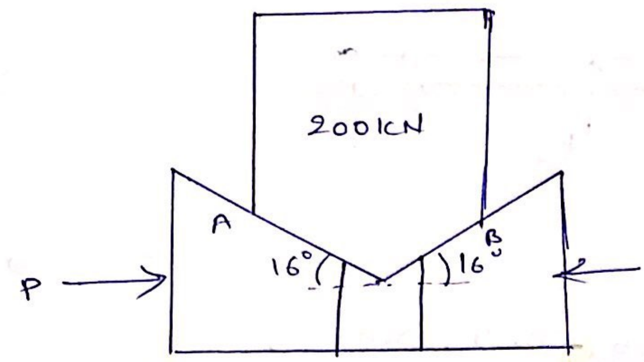
$\Sigma H = 0$

$-P + 0.3R_3 + 0.3R_2\cos 20^\circ + R_2 \sin 20^\circ = 0$

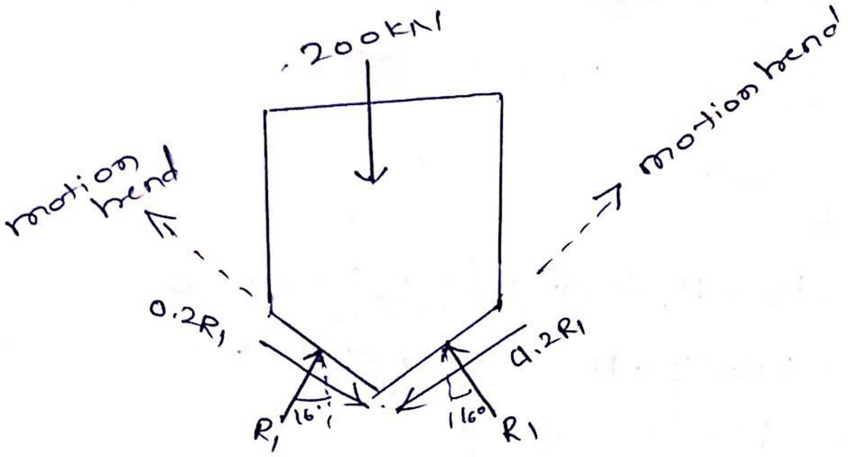
$-P + 0.3 \times 12.80 + 0.3 \times 15.29\cos 20^\circ + 15.29 \sin 20^\circ = 0$

$P = 13.34 \text{ kN}$

2) A body of weight 200 kN is to be raised by means of the same wedges A & B as shown in fig below. find the force P for impending motion of block C upwards, if the co-efficient of friction 0.2 for all contact surfaces.



Soln:-



FBD of body

$\Sigma H = 0$

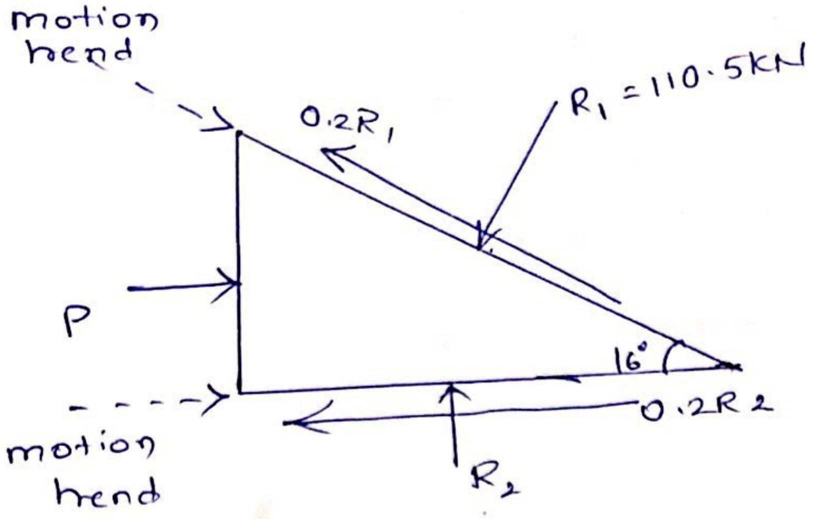
$0.2R_1 \cos 16° + R_1 \sin 16° - 0.2R_1 \cos 16° - R_1 \sin 16° = 0$

$\Sigma V = 0$

$-200 + 2R_1 \cos 16° - 2 \times 0.2 \times R_1 \sin 16° = 0$

$R_1 = 110.5 kN.$

FBD of Wedge A



$\Sigma V = 0$

$R_2 - R_1 \cos 16° + 0.2R_1 \sin 16° = 0$

$R_2 - 110.5 \cos 16° + 0.2 \times 110.5 \sin 16° = 0$

$R_2 = 100.13 \text{KN}$

$\Sigma H = 0$

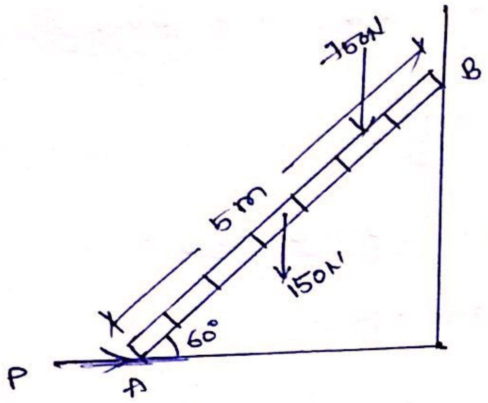
$P - 0.2 R_2 - R_1 \sin 16° - 0.2 R_1 \cos 16° = 0$

$P - 0.2 \times 100.13 - 110.5 \sin 16° - 0.2 \times 110.5 \cos 16° = 0$

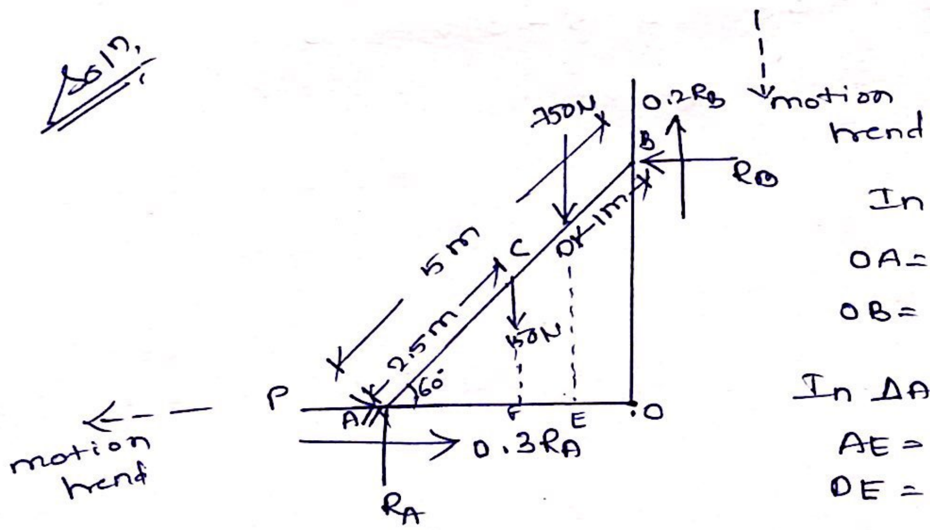
$P = 73.73 \text{KN}.$

Problems On Ladder friction

1) A ladder weighing 250N & length 5 m is placed against a vertical wall as shown. The co-efficient of friction between ladder & wall is 0.2 & between ladder & floor is 0.3. The ladder has to support a man weighing 750N @ a distance of 1m along the ladder from wall. Calculate the minimum horizontal force to be applied to A to prevent the slipping.



Soln,



In ΔAOB,

$OA = 5\cos 60° = 2.5m$

$OB = 5\sin 60° = 4.33m$

In ΔAEB

$AE = 4\cos 60° = 2m$

$DE = 4\sin 60°$

In $\Delta^{le} AFC$

$AF = 2.5\cos 60° = 1.25m$

$CF = 2.5\sin 60°$

$\Sigma V = 0$

$R_A - 250 - 750 + 0.2 R_B = 0$ —— (1)

$\Sigma H = 0$

$P + 0.3 R_A - R_B = 0$ —— (2)

$\Sigma M_A = 0$

$-R_B \times OB - 0.2 R_B \times OA + 250 \times AF + 750 \times AE = 0$

$-R_B \times 4.33 - 0.2 \times R_B \times 2.5 + 250 \times 1.125 + 750 \times 2 = 0$

$R_B = 375.26 N$

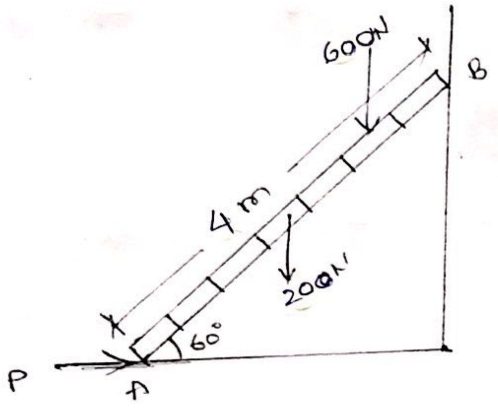
(1) $\Rightarrow$ $R_A - 250 - 750 + 0.2 \times 375.26 = 0$

$R_A = 924.95 N$

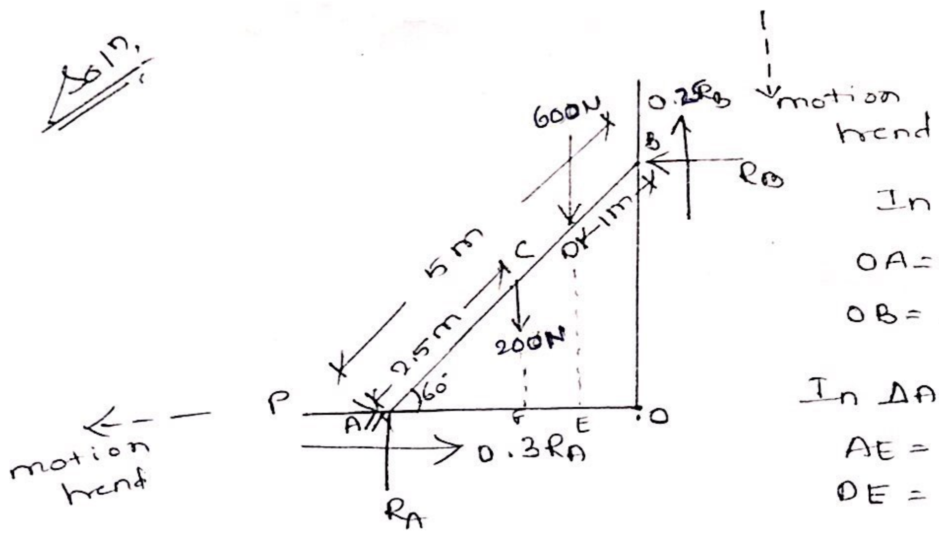
(2) $\Rightarrow$ $P + 0.3 \times 924.95 - 375.26 = 0$

$P = 97.78 N$

2) A ladder weighing 200N & length 4m is placed against a vertical wall as shown. The co-efficient of friction between ladder & wall is 0.25 & between ladder & floor is 0.3. The ladder has to support a man weighing 600N @ a distance of 1m along the ladder from wall. Calculate the minimum horizontal force to be applied to A to prevent the slipping.



Soln,



In ΔAOB,

OA = 4 cos60° = 2 m

OB = 4 sin60° = 3.46m

In ΔAED

AE = 3cos60° = 1.5m

DE = 3 sin60°

In Δ^{le} AFC

AF = 2 cos60° = 1m

CF = 2 sin60°

$\Sigma V = 0$

$R_A - 200 - 600 + 0.25 R_B = 0$ —— (1)

$\Sigma H = 0$

$P + 0.3 R_A - R_B = 0$ —— (2)

$\Sigma M_A = 0$

$-R_B \times OB - 0.25 R_B \times OA + 200 \times AF + 600 \times AE = 0$

$-R_B \times 3.46 - 0.25 \times R_B \times 2 + 200 \times 1 + 600 \times 1.5 = 0$

$R_B = 277.78\ N$

(1) $\Rightarrow$ $R_A - 200 - 600 + 0.25 \times 277.78 = 0$

$R_A = 730.56\ N$

(2) $\Rightarrow$ $P + 0.3 \times 730.56 - 277.78 = 0$

$P = 58.61\ N$