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$\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$

S_1 = Sum of diagonal Elements

S_2 = Sum of Minors of diagonal Elements

S_3 = Adjoint of A

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 1 \\ 1 & 6 & 2 \end{bmatrix}$$

$= 1(8-6) - 2(2-1) + 3(6-4)$

$2 - 2 + 6$

$= 6$

$S_1 = 7$

$\lambda^3 - 7\lambda^2 + 3\lambda - 6 = 0$

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(1) $y = (ax+b)^m$

$y_1 = (ax+b)^{m-1} \cdot a$

$y_2 = m(m-1)(ax+b)^{m-2} \cdot a \cdot a$

$= m(m-1)(ax+b)^{m-2} a^2$

$y = e^{ax}$

$y_1 = ae^{ax}$

$y_2 = a^2 e^{ax}$

$y_3 = a^3 e^{ax}$

$y_n = a^n e^{ax}$

$y_3 = m(m-1)(m-2)(ax+b)^{m-3} a^3$

$y_4 = m(m-1)(m-2)(m-3)(ax+b)^{m-4} a^4$

$y_n = m(m-1)(m-2) \ldots (m-(n-1))(ax+b)^{m-n} \cdot a^n$

$m-(n-1)$

$m-(3-1)$

(2) $y = \sin(ax+b)$

$y_1 = \cos(ax+b) \cdot a$

$\cos\theta = \sin(\pi/2 + \theta)$

$y_1 = \sin(\pi/2 + ax + b) \cdot a$

$y_2 = \cos(\pi/2 + ax + b) \cdot a^2$

$y_2 = \sin\left(2\frac{\pi}{2} + ax + b\right) \cdot a^2$

$y_3 = \sin\left(3\frac{\pi}{2} + ax+b\right) . a^3$

$y_4 = \sin\left(4\frac{\pi}{2} + ax+b\right) . a^4$

$\vdots$

$y_n = \sin\left(\frac{n\pi}{2} + ax+b\right) . a^n$

(3) $y = \cos(ax+b)$

$y_1 = -\sin(ax+b) . a$

$-\sin\theta = \cos(\pi/2 - \theta)$

$y_1 = \cos \cancel{\pi/2 + a} (\pi/2 - \theta + ax+b) . a$

$y_2 = -\sin((\pi/2 - \theta) + ax+b) . a^2$

$y_2 = \cos\left(2\left(\frac{\pi}{2} - \theta\right) + ax+b\right) . a^2$

$y_3 = \cos\left(3\left(\frac{\pi}{2} - \theta\right) + ax+b\right) . a^3$

$y_4 = \cos\left(4\left(\frac{\pi}{2} - \theta\right) + ax+b\right) . a^4$

$\vdots$

$y_n = \cos\left(\frac{n\pi}{2} + ax+b\right) . a^n$

Leibnitz theorem

$$D^n(uv) = (uv)_n = u v_n + {}^nC_1\, u_1 v_{n-1} + {}^nC_2\, u_2 v_{n-2} + \cdots + u_n v$$

Ex.

$y = x^2 e^x$	$v = e^x$
$u = x^2$	$v_1 = e^x$
$u_1 = 2x$	$v_2 = e^x$
$u_2 = 2$	
$u_3 = 0$	
$u_4 = 0$	$v_{n-1} = e^x$
	$v_n = e^x$
$u_n = 0$	

$$D^n(x^2 e^x) = x^2 e^x + {}^nC_1\, 2x + {}^nC_1\, 2x e^x + {}^nC_2\, 2 e^x + {}^nC_3\, 0 e^x$$
$$= x^2 e^x + 2\,{}^nC_1\, x e^x + 2\,{}^nC_2\, e^x$$

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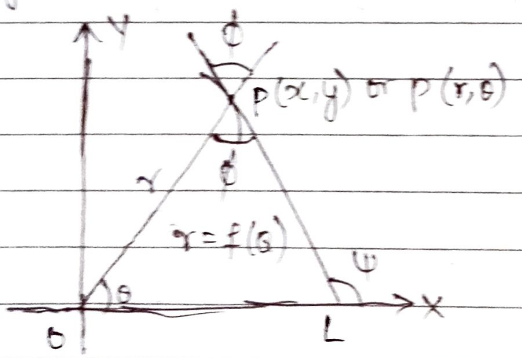
★ Angle between Radius vector & Tangent



$\psi = \phi + \theta \rightarrow (1)$

$\angle XOP = \theta$ $\quad\quad$ $\angle OPL = \phi$

$\angle PLX = \psi$, $\quad$ $OP = r$

$\tan\psi = \tan(\phi + \theta)$

Halsur

$$= \frac{\tan\phi + \tan\theta}{1 - \tan\phi \cdot \tan\theta} \longrightarrow (2)$$

$x = r\cos\theta, \quad y = r\sin\theta$

$$\tan\psi = \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$$

$$\frac{dx}{d\theta} = -r\sin\theta + \cos\theta\frac{dx}{d\theta}$$

$$\frac{dy}{d\theta} = r\cos\theta + \sin\theta\frac{dx}{d\theta}$$

lets assume $\frac{dx}{d\theta} = r'$

$$\tan\psi = \frac{r\cos\theta + r'\sin\theta}{-r\sin\theta + r'\cos\theta}$$

$\div$ num of deno by $r'\cos\theta$

$$\tan\psi = \frac{\frac{r\cos\theta}{r'\cos\theta} + \frac{r'\sin\theta}{r'\cos\theta}}{\frac{r\sin\theta}{r'\cos\theta} + \frac{\cancel{r'\cos\theta}}{\cancel{r'\cos\theta}}}$$

$$= \frac{\frac{r}{r'} + \tan\theta}{-\frac{r}{r'}\tan\theta + 1}$$

$$= \frac{\frac{r}{r'} + \tan\theta}{1 - \frac{r}{r'}\tan\theta} \longrightarrow (3)$$

$$\tan\phi = \frac{r}{r'} = \frac{r}{\frac{dr}{d\theta}}$$

$$\boxed{\tan\phi = r\left(\frac{d\theta}{dr}\right)}$$

$$\boxed{\cot\phi = \frac{1}{r\left(\frac{d\theta}{dr}\right)} = \frac{1}{r}\left(\frac{dr}{d\theta}\right)}$$

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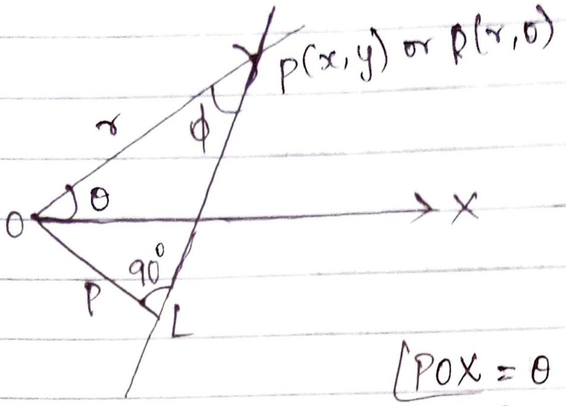
* Length of perpendicular from the pole to the tangent :-

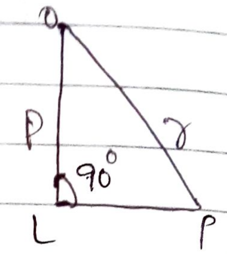
(or) (Pedal Equation) :-



$\angle POX = \theta$, $OP = r$

$\angle OLP = 90^\circ$, $OL = p$

$\angle OPL = \phi$



$$\sin\theta = \frac{OL}{OP}$$

$$\sin\theta = \frac{p}{r}$$

$$\Rightarrow \boxed{p = r\sin\theta} \rightarrow (1)$$

$$\cot\phi = \frac{1}{r}\left(\frac{dr}{d\theta}\right) \rightarrow (2)$$

Sq. on b.s & taking reciprocal

$$\frac{1}{p^2} = \frac{1}{r^2}\left(\frac{1}{\sin^2\theta}\right)$$

$$\frac{1}{p^2} = \frac{1}{r^2}\left(\operatorname{cosec}^2\theta\right)$$

$$\frac{1}{p^2} = \frac{1}{r^2}\left(1 + \cot^2\theta\right)$$

$$\frac{1}{p^2} = \frac{1}{r^2}\left[1 + \frac{1}{r^2}\left(\frac{dr}{d\phi}\right)^2\right]$$

$$\boxed{\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r} + \left(\frac{dr}{d\phi}\right)^2} \rightarrow (3)$$

put $\frac{1}{r} = u$

diff w.r.t θ

$$-\frac{1}{r^2}\frac{dr}{d\theta} = \frac{du}{d\theta}$$

Squaring on b.s

$$\frac{1}{r^4}\left(\frac{dr}{d\theta}\right)^2 = \left(\frac{du}{d\theta}\right)^2 \longrightarrow \text{sub } \phi \text{ in } (3)$$

$$\boxed{\frac{1}{p^2} = u^2 + \left(\frac{du}{d\theta}\right)^2}$$

Angle of intersection of two polar curves:-

$$\tan\phi_1 \cdot \tan\phi_2 = -1$$

1) $\sin\left(\frac{\pi}{2}-\theta\right) = \cos\theta$, $\cos\left(\frac{\pi}{2}-\theta\right) = \sin\theta$

$\tan\left(\frac{\pi}{2}-\theta\right) = \cot\theta$, $\cot\left(\frac{\pi}{2}-\theta\right) = \tan\theta$

2) $\sin\left(\frac{\pi}{2}+\theta\right) = \cos\theta$, $\cos\left(\frac{\pi}{2}-\theta\right) = -\sin\theta$

$\tan\left(\frac{\pi}{2}+\theta\right) = -\cot\theta$, $\cot\left(\frac{\pi}{2}+\theta\right) = -\tan\theta$

3) $\tan\left(\frac{\pi}{4}+\theta\right) = \frac{1+\tan\theta}{1-\tan\theta}$, $\cot\left(\frac{\pi}{4}+\theta\right) = \frac{1-\tan\theta}{1+\tan\theta}$

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$1 + \cos\theta = 2\cos^2 \theta/2$, $1 - \cos\theta = 2\sin^2 \theta/2$

$\sin\theta = 2\sin\theta/2 \cos\theta/2$, $\cos\theta = \cos^2\theta/2 - \sin^2\theta/2$

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$|\phi_1 - \phi_2| = \pi/2$

Find the angle between the radius vector and the tangent for the following polar curves.

1) $r = a(1-\cos\theta)$

$\log r = \log [a(1-\cos\theta)]$

$\log r = \log a + \log(1-\cos\theta)$

$$\frac{1}{r}\frac{dr}{d\theta} = 0 + \frac{1}{1-\cos\theta}(\sin\theta)$$

$$= \frac{\cancel{2}\cancel{\sin\theta/2}\cos\theta/2}{\cancel{2}\sin^{\cancel{2}}\theta/2} \Rightarrow \frac{\cos\theta/2}{\sin\theta/2}$$

$$\cot\phi = \cot\theta/2$$

$$\Rightarrow \boxed{\phi = \theta/2}$$

2) $r^2 \cos 2\theta = a^2$

$2\log r + \log(\cos 2\theta) = 2\log a$

diff w.r.t 'θ'

$$2\frac{1}{r}\frac{dr}{d\theta} + \frac{1}{\cos 2\theta} - \sin 2\theta \cdot 2 = 0$$

$$\frac{2}{r}\frac{dr}{d\theta} = \frac{2\sin 2\theta}{\cos 2\theta} = 0$$

$$\frac{1}{r}\frac{dr}{d\theta} = \tan 2\theta$$

$$\cot\phi = \cot\left(\frac{\pi}{2} - 2\theta\right)$$

$$\boxed{\phi = \frac{\pi}{2} - 2\theta}$$

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3) $r^m = a^m(\cos m\theta + \sin m\theta)$

$m\log r = m\log a + \log(\cos m\theta + \sin m\theta)$

$m\frac{1}{r}\frac{dr}{d\theta} = 0 + \frac{1}{\cos m\theta + \sin m\theta}\left[-m\sin m\theta + m\cos m\theta\right]$

$m\frac{1}{r}\frac{dr}{d\theta} = \frac{m\left[\cos m\theta - \sin m\theta\right]}{\cos m\theta + \sin m\theta}$

$= \frac{\cancel{\cos m\theta}\left[1 - \frac{\sin m\theta}{\cos m\theta}\right]}{\cancel{\cos m\theta}\left[1 + \frac{\sin m\theta}{\cos m\theta}\right]}$

$\cot\phi = \frac{1 - \tan m\theta}{1 + \tan m\theta} = \cot\left(\frac{\pi}{4} + m\theta\right)$

$\boxed{\phi = \frac{\pi}{4} + m\theta}$

4) $\frac{1}{r} = 1 + e\cos\theta$

$\log 1 - \log r = \log(1 + e\cos\theta)$

$0 - \frac{1}{r}\frac{dr}{d\theta} = \frac{1}{1 + e\cos\theta}(-e\sin\theta)$

$\cot\phi = \frac{e\sin\theta}{1 + e\cos\theta}$

$\phi = \cot^{-1}\left(\frac{e\sin\theta}{1 + e\cos\theta}\right)$

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Find the angle b/w the radius vector and the tangent and also find the slope of the tangent as indicated for the following curves.

i) $r = a(1+\cos\theta)$ at $\theta = \pi/3$

$\log r = \log a + \log(1+\cos\theta)$

$\frac{1}{r}\frac{dr}{d\theta} = 0 + \frac{1}{1+\cos\theta}(-\sin\theta)$

$\cot\phi = \frac{-\cancel{2}\sin\theta/2 \cos\theta/2}{\cancel{2}\cos^2\theta/2}$

$\cot\phi = -\tan\theta/2$

$= \cot\left(\frac{\pi}{2} + \theta/2\right)$

$$\boxed{\phi = \frac{\pi}{2} + \frac{\theta}{2}}$$

$\phi = \frac{\pi}{2} + \frac{\pi}{6} \Rightarrow \phi = \frac{2\pi}{3}$

$\psi = \theta + \phi = \frac{\pi}{3} + \frac{2\pi}{3} = \pi$

$\tan\psi = \tan\pi = 0$

ii) $r\cos^2(\theta/2) = a$ at $\theta = 2\pi/3$

$\log r + 2\log(\cos\theta/2) = \log a \Rightarrow \frac{1}{r}\frac{dr}{d\theta} + 2\frac{1}{\cos\theta/2}(-\sin\theta/2)\frac{1}{2} = 0$

$\frac{1}{r}\frac{dr}{d\theta} = \frac{\cancel{2}\sin\theta/2}{\cos\theta/2} = \frac{1}{\cancel{2}}$

$\cot\phi = \tan\theta/2$

$= \cot(\pi/2 - \theta/2)$

$\phi = (\pi/2 - 2\pi/3 / 2)$

$$\phi = \left(\frac{\pi}{2} - \frac{2\pi}{6}\right)$$

$$\phi = \left(\frac{\pi}{2} - \frac{\pi}{3}\right)$$

$$\phi = \frac{\pi}{6}$$

$$\psi = \phi + \theta = \pi/6 + 2\pi/3 = 5\pi/6$$

$$\tan(150) = -0.577$$

iii) $\frac{2a}{r} = (1-\cos\theta)$ at $\theta = 2\pi/3$

$$\log 2a - \log r = \log(1-\cos\theta)$$

$$-\frac{1}{r}\frac{dr}{d\theta} = \frac{1}{1-\cos\theta} - \sin\theta$$

$$\frac{1}{r}\frac{dr}{d\theta} = \frac{+\sin\theta}{1-\cos\theta}$$

$$= \frac{2\sin\theta/2 \cdot \cos\theta/2}{2\sin^2\theta/2}$$

$$\cot\phi = \pi/2$$

$$\phi = 2\pi/3/2 = -\pi/3$$

$$\phi = -\pi/3$$

$$\phi = \phi + \theta$$

$$= -\pi/3 + 2\pi/3$$

$$= \frac{2\pi - \pi}{3}$$

$$\psi = \pi/3$$

$$\tan\psi = \tan(\pi/3) = \sqrt{3}$$

iv) $r = a(1+\sin\theta)$ at $\theta = \pi/2$

$\log r = \log a + \log(1+\sin\theta)$

diff w.r.t 'θ'

$$\frac{1}{r}\frac{dr}{d\theta} = 0 + \frac{\cos\theta}{1+\sin\theta}$$

$$\cot\phi_1 = \frac{\cos\theta}{1+\sin\theta}$$

$$\theta = \pi/2, \quad \cot\phi = \frac{0}{1+1} = 0$$

$$\cot\phi = 0 \Rightarrow \boxed{\phi = \pi/2}$$

$$\psi = \theta + \phi$$
$$= \frac{\pi}{2} + \frac{\pi}{2} = \pi$$

$\Rightarrow$ $r = a(1+\cos\theta)$

$\log r = \log a + \log(1+\cos\theta)$

$$\frac{1}{r}\frac{dr}{d\theta} = 0 + \frac{1}{1+\cos\theta}(-\sin\theta)$$

$$\cot\phi_1 = \frac{-\cancel{2}\sin\theta/2\ \cos\theta/2}{\cancel{2}\cos^2\theta/2}$$

$$\cot\phi_1 = -\tan\theta/2$$
$$\cot\phi_2 = \cot(\pi/2 + \theta/2)$$
$$\phi_1 = \frac{\pi}{2} + \frac{\theta}{2}$$

$r = b(1-\cos\theta)$

$\log r = \log b + \log(1-\cos\theta)$

$$\frac{1}{r}\frac{dr}{dt} = 0 + \frac{1}{1-\cos\theta}(\sin\theta)$$

$$\cot\phi_2 = \frac{\cancel{2}\sin\theta/2\ \cos\theta/2}{\cancel{2}\sin^2\theta/2}$$

$$\cot\phi_2 = \cot\theta/2$$

$$\boxed{\phi_2 = \theta/2}$$

$$|\phi_1 - \phi_2| = |\pi/2 + \theta/2 - \theta/2|$$
$$= \pi/2$$

$\Rightarrow$ $r = a(1+\sin\theta)$

$\log r = \log a + \log(1+\sin\theta)$

$\frac{1}{r}\frac{dr}{d\theta} = 0 + \frac{1}{1+\sin\theta}(\cos\theta)$

$\cot\phi_1 = \frac{\cos\theta}{1+\sin\theta}$

$\tan\phi_1 = \frac{1+\sin\theta}{\cos\theta}$

$\tan\phi_1 \cdot \tan\phi_2 = \frac{1-\sin^2\theta}{-\cos^2\theta}$

$= \frac{\cos^2\theta}{-\cos^2\theta} = -1$

$r = b(1-\sin\theta)$

$\log r = \log b + \log(1-\sin\theta)$

$\frac{1}{r}\frac{dr}{d\theta} = 0 + \frac{1}{1-\sin\theta}(\cos\theta)$

$\cot\phi_2 = \frac{\cos\theta}{1-\sin\theta}$

$\tan\phi_2 = \frac{(1-\sin\theta)}{-\cos\theta}$

~~$\tan\phi_2 =$~~

$\Rightarrow$ $r^n = a^n \cos m\theta$

$n\log r = n\log a + \log(\cos m\theta)$

$n\frac{1}{r} = 0 + \frac{1}{(\cos m\theta)}$

$n\frac{1}{r}\frac{dr}{d\theta} = \frac{n(\cos m\theta)}{\cos m\theta}$

$\cot\phi = \frac{-n\sin n\theta}{\cos n\theta}$

$\cot\phi = \cot(\pi/2 + m\theta)$

$\phi_1 = \pi/2 + m\theta$

$r^n = b^n \sin n\theta$

$n\log r = n\log b + \log(\sin n\theta)$

$n\frac{1}{r} = 0 + \frac{1}{(\sin n\theta)}$

$n\frac{1}{r}\frac{dr}{d\theta} = 0 + \frac{1}{\sin\theta}(\cos n\theta)$

$\frac{n}{r}\frac{dr}{dt} = \frac{n\cos n\theta}{\sin\theta}$

$\cot\phi_2 = \cot n\theta$

$\phi_2 = n\theta$

$|\phi - \phi_2| = \pi/2$

$|\pi/2 + n\theta - n\theta| = \pi/2$

$\pi/2 = \pi/2$

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$\Rightarrow$ $r = 4\sec^2 \theta/2$ $\qquad$ $r = 9\operatorname{cosec}^2 \theta/2$

$\phi_1 = \pi/4$ $\qquad$ $\phi_2 = -\pi/4$

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Find the angle of intersection of the following pairs of curves :-

(1)

$r = \sin\theta + \cos\theta$

$\log r = \log(\sin\theta + \cos\theta)$

$\frac{1}{r}\frac{dr}{d\theta} = \frac{1}{(\sin\theta+\cos\theta)}(\cos\theta - \sin\theta)$

$\cot\phi_1 = \frac{\cos\theta - \sin\theta}{\sin\theta + \cos\theta}$

$\frac{\cancel{\cos\theta}(1-\tan\theta)}{\cancel{\cos\theta}(1+\tan\theta)}$

$\cancel{\cot}\phi_1 = \cancel{\cot}(\pi/4 + \theta)$

$\boxed{\phi_1 = \frac{\pi}{4} + \theta}$

$r = 2\sin\theta$

$\log r = \log 2 + \log \sin\theta$

$\frac{1}{r}\frac{dr}{d\theta} = 0 + \frac{1}{\sin\theta}\cos\theta$

$\cancel{\cot}\phi_2 = \cancel{\cot}\theta$

$\boxed{\phi_2 = \theta}$

$|\phi_1 - \phi_2| = |\pi/4 + \cancel{\theta} - \cancel{\theta}|$

$= \frac{\pi}{4} = 45^\circ$

$\frac{1}{\log\theta}$

(2)

$r = a(\log\theta)$

$\log r = \log a + \log(\log\theta)$

diff w.r.t 'θ'

$\frac{1}{r}\frac{dr}{d\theta} = 0 + \frac{1}{\log\theta}\frac{1}{\theta}$

$\cot\phi_1 = \frac{1}{\theta\log\theta}$

$\tan\phi_1 = \theta\log\theta$

$\tan(\phi_1 - \phi_2) = \frac{\tan\phi_1 + \tan\phi_2}{1 + \tan\phi_1 \cdot \tan\phi_2}$

$r = \frac{a}{\log\theta}$

$\log r = \log a - \log(\log\theta)$

$\frac{1}{r}\frac{dr}{d\theta} = 0 - \frac{1}{\log\theta}\frac{1}{\theta}$

$\cot\phi_2 = \frac{-1}{\theta\log\theta}$

$\tan\phi_2 = -\theta\log\theta$

$$= \frac{\theta \log\theta + \theta \log\theta}{1 + \theta \log\theta(-\theta \log\theta)}$$

$$= \frac{2\theta \log\theta}{1 - (\theta \log\theta)}$$

$$\cancel{\theta} \log\theta = \frac{\cancel{\theta}}{\log\theta}$$

$$\log\theta \log\theta = 1$$

$$(\log\theta)^2 = 1$$

$$\boxed{\theta = e}$$

$$\tan(\phi_1 - \phi_2) = \frac{2e \log e}{1 - (e \log e)^2} = \frac{2e}{1 - e^2}$$

$$\phi_1 - \phi_2 = \tan^{-1}\left(\frac{2e}{1-e^2}\right)$$

(3) $r = a(1 - \cos\theta)$ and $r = 2a\cos\theta$

$\log r = \log a + \log(1 - \cos\theta)$

$\log r = \log 2a + \log \cos\theta$

$\frac{1}{r}\frac{dr}{d\theta} = 0 + \frac{1}{1-\cos\theta}(\sin\theta)$

$\frac{1}{r}\frac{dr}{d\theta} = 0 + \frac{1}{\cos\theta}(-\sin\theta)$

$\cot\phi_1 = \frac{\cancel{2}\cancel{\sin\theta/2}\cos\theta/2}{\cancel{2}\sin^2\theta/2}$

$\cot\phi_2 = -\tan\theta$

$\cot\phi_1 = \cot\theta/2$

$\cot\phi_2 = \cot\left(\frac{\pi}{2} + \theta\right)$

$\boxed{\phi_1 = \frac{\theta}{2}}$

$\boxed{\phi_2 = \frac{\pi}{2} + \theta}$

$$a(1 - \cos\theta) = 2a\cos\theta$$

$$a - a\cos\theta = 2a\cos\theta$$

$$3\cancel{a}\cos\theta = \cancel{a}$$

$$3\cos\theta = 1$$

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$\cos\theta = 1/3$

$\boxed{\theta = \cos^{-1}(1/3)}$

$|\phi_1 - \phi_2| = \cancel{\phi}\, |\theta/2 - \pi/2 - \theta|$

$= \frac{\cos^{-1}(1/3)}{2} - \frac{\pi}{2} - \cos^{-1}(1/3)$

(4) $r = 6\cos\theta$

$\log r = \log 6 + \log\cos\theta$

$\frac{1}{r}\frac{dr}{d\theta} = \frac{1 - \sin\theta}{\cos\theta}$

$\cot\phi = -\tan\phi$

$\cancel{\cot\phi} = \cot(\pi/2 + \theta)$

$\phi_1 = \pi/2 + \theta$

$r = 2(1 + \cos\theta)$

$\log r = \log 2 + \log(1 + \cos\theta)$

diff w.r.t 'θ'

$\frac{1}{r}\frac{dr}{d\theta} = 0 + \frac{\log}{(1 + \cos\theta)}$

$\frac{1}{r}\frac{dr}{d\theta} = \frac{1}{1 + \tan\theta}\quad 1 - \sin\theta$

$\cot\phi_2 = -\frac{\cancel{2}\sin\theta/2 \cos\theta/2}{\cancel{2}\cos^2\theta/2}$

$\cot\phi_2 = -\tan\theta/2$

$\phi_2 = \frac{\pi}{2} + \frac{\theta}{2}$

$(\phi_1 - \phi_2) = \pi/2 + \theta + \pi/2 + \theta/2$

$|\phi_1 - \phi_2| = \frac{\cos^{-1}(1/2)}{2}$

$\frac{\cos^{-1}(1/2)}{2}$

$\frac{60}{2}$

$= \cancel{\pi/3}$

$= \pi/6$

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Pedal Equation

(1) $\frac{2a}{r} = 1 + \cos\theta$

$\log 2a - \log r = \log(1 + \cos\theta)$

$0 - \frac{1}{r}\frac{dr}{d\theta} = \frac{1}{1+\cos\theta}(-\sin\theta)$

$\cancel{-} \cot\phi = \frac{\cancel{-}\cancel{2}\sin\theta/2 \cdot \cancel{\cos\theta/2}}{\cancel{2}\cos^{\cancel{2}}\theta/2}$

$\cot\phi = \tan\theta/2$

$\cancel{\cot}\,\phi = \cancel{\cot}\,(\pi/2 - \theta/2)$

$\boxed{\phi = \pi/2 - \theta/2}$

$P = r\sin\phi$

$P = r\sin\left(\frac{\pi}{2} - \frac{\theta}{2}\right)$

$P = r\cos\theta/2$

$\frac{P}{r} = \cos\theta/2 \Rightarrow$ Sq. on b.s $\Rightarrow \frac{P^2}{r^2} = \cos^2\theta/2$

$\frac{2a}{r} = 1 + \cos\theta$

$\frac{\cancel{2}a}{r} = \cancel{2}\cos^2\theta/2$

$\frac{a}{\cancel{r}} = \frac{p^2}{r^{\cancel{2}}}$

$\Rightarrow \boxed{P^2 = ar}$

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(2) $r(1-\cos\theta) = 2a$

$\log r + \log(1-\cos\theta) = \log 2a$

$$\frac{1}{r}\frac{dr}{d\theta} + \frac{1}{1-\cos\theta} \cdot \sin\theta = 0$$

$$\cot\phi = -\frac{\cancel{2}\sin\theta/2\cos\theta/2}{\cancel{2}\sin^2\theta/2}$$

$$\cot\phi = -\cot\theta/2$$

$$\boxed{\phi = -\theta/2}$$

$$P = r\sin\phi$$
$$= r\sin(-\theta/2)$$
$$P = -r\sin\theta/2$$

$$r(1-\cos\theta) = 2a$$
$$r(\cancel{2}\sin^2\theta/2) = \cancel{2}a$$
$$\frac{a}{r} = \sin^2\theta/2$$

$$\frac{p^2}{r^{\cancel{2}}} = \frac{a}{\cancel{r}}$$

$$\boxed{p^2 = ar}$$

(3) $r^2 = a^2\sec 2\theta$

$2\log r + 2\log a + \log(\sec 2\theta)$

$$\frac{2}{r}\frac{dr}{d\theta} = \frac{1}{\sec 2\theta} \sec\theta\tan\theta \cdot 2$$

$$\frac{1}{r}\frac{dr}{d\theta} = \frac{\sec^2\theta\tan^2\theta}{\sec 2\theta}$$

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$$\cot\phi = \frac{\sec^2\theta \cdot \tan^2\theta}{\sec^2\theta}$$

$$\cot\phi = \cot(\pi/2 - 2\theta)$$

$$\phi = (\pi/2 - 2\theta)$$

$$\frac{r^2}{a^2} = \sec 2\theta$$

$$p = r\sin r$$

$$= r\sin(\pi/2 - 2\theta)$$

$$p = r\cos(2\theta)$$

$$\frac{p}{r} = \cos 2\theta$$

$$\frac{a^2}{r^2} = \cos 2\theta$$

$$\frac{p}{\cancel{r}} = \frac{a^2}{r^{\cancel{2}}}$$

$$p = a^2/r$$

$$a^2 = pr$$

(4)

$$r^n = a^n(\cos n\theta)$$

$$n\log r = n\log a + \log(\cos n\theta)$$

$$n \times \frac{1}{r} \cdot \frac{dr}{d\theta} = 0 + \frac{1}{\cos n\theta}(-\sin n\theta) \, n$$

$$\cot\phi = -\frac{\sin n\theta}{\cos n\theta}$$

$$\cot\phi = -\tan\theta$$

$$\cot\phi = \cot(\pi/2 + n\theta)$$

$$\boxed{\phi = \pi/2 + n\theta}$$

$$p = r\sin\phi$$

$$= r\sin(\pi/2 + n\theta)$$

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$$p = r\cos n\theta$$

$$r^n = a^n(\cos n\theta)$$

$$\frac{p}{r} = \frac{r^n}{a^n}$$

$$\boxed{pa^n = r^{n+1}}$$

(5) $r^m = a^m(\cos m\theta + \sin m\theta)$

$m\log r = m\log a + \log(\cos m\theta + \sin m\theta)$

$= m \times \frac{1}{r}\frac{dr}{d\theta} = 0 + \frac{1}{(\cos m\theta + \sin m\theta)}(\sin m\theta - \cos m\theta)$

$$\phi = \frac{\pi}{4} + m\theta$$

$$p = r\sin\phi = r\sin(\pi/4 + m\theta)$$

$$= r\left[\sin\frac{\pi}{4}\cos m\theta + \cos\frac{\pi}{4}\sin m\theta\right]$$

$$p = \frac{r}{1}(1/\sqrt{2}\cos m\theta + 1/\sqrt{2}\sin m\theta)$$

$$p = \frac{r}{\sqrt{2}}(\cos m\theta + \sin m\theta)$$

$$\frac{p\sqrt{2}}{r} = \cos m\theta + \sin m\theta$$

$$r^m = a^m\left[\frac{p\sqrt{2}}{r}\right]$$

$$\boxed{r^{m+1} = \sqrt{2}\,a^m p}$$

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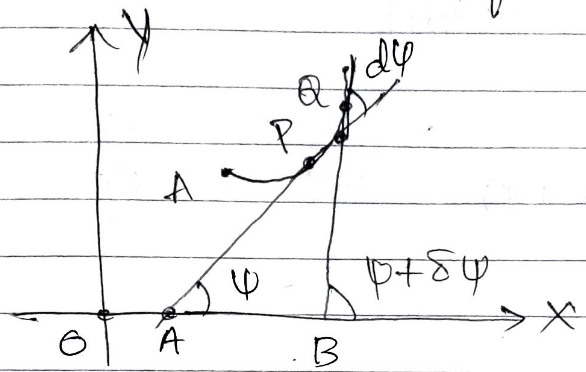
Curvature and Radius of Curvature



$\hat{AP} = s$

$\hat{PQ} = \delta s$

$\hat{AQ} = s + \delta s$

$$\boxed{K = \frac{d\psi}{ds}}$$

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Radius of Curvature

$$\rho = \frac{(1+y_1^2)^{3/2}}{y_2}$$

$$S = \frac{ds}{d\psi} \qquad \rho = \frac{(1+x_1^2)^{3/2}}{x_2}$$

① Find the radius of curvature for the curve whose intrinsic equation is $S = a \log \tan\left(\frac{\pi}{4} + \frac{\psi}{2}\right)$

diff w.r.t ψ

$$\frac{ds}{d\psi} = a \frac{1}{\tan(\pi/4 + \psi/2)} \cdot \sec^2(\pi/4 + \psi/2) \cdot 1/2$$

$\left(\frac{1}{\tan} = \cot\right)$

$\cot = \frac{\cos}{\sin}$

$$= \frac{a}{2} \cdot \frac{\cos(\pi/4 + \psi/2)}{\sin(\pi/4 + \psi/2)} \cdot \frac{1}{\cos^2(\pi/4 + \psi/2)}$$

$$= \frac{a}{\sin\left(2\left(\frac{\pi}{4} + \psi/2\right)\right)}$$

$$= \frac{a}{\sin\left(\frac{\pi}{2} + \psi\right)} = \frac{a}{\cos\psi} = \frac{ds}{d\psi}$$

② Find the radius of curvature for $y = a \log \sec(x/a)$

$$y = a \log \sec(x/a)$$

$$y_1 = \frac{dy}{dx} = a \frac{1}{\sec(x/a)} \cdot \sec(x/a) \tan(x/a) \cdot \frac{1}{a}$$

$$y_1 = \tan\left(\frac{x}{a}\right)$$

$$y_2 = \sec^2\left(\frac{x}{a}\right) \cdot \frac{1}{a}$$

$$y_2 = \frac{1}{a} \sec^2\left(\frac{x}{a}\right)$$

$$\rho = \frac{(1 + y_1^2)^{3/2}}{y_2}$$

$$\rho = \frac{1 + \tan^2(x/a)^{3/2}}{\frac{1}{a} \sec^2(x/a)}$$

$$= \frac{a\left(\sec^2(x/a)\right)^{3/2}}{\sec^2(x/a)}$$

$$= \frac{a \sec^3(x/a)}{\sec^2(x/a)} = a \sec(x/a)$$

(3) $y = c\cosh\left(\frac{x}{c}\right)$

$y_1 = \frac{dy}{dx} = c\ \overset{1}{\overline{\sinh}}\left(\frac{x}{c}\right)$

$y_1 = \sinh\left(\frac{x}{c}\right)$

$y_2 = \cosh\left(\frac{x}{c}\right)\cdot\left(\frac{1}{c}\right)$

$y_2 = \frac{1}{c}\cancel{\cos}\sin\left(\frac{x}{c}\right)$

$$\rho = \frac{\left(1+y_1^2\right)^{3/2}}{y_2}$$

$$\rho = \frac{1+\sinh^2(x/c)^{3/2}}{\frac{1}{c}\cos\left(\frac{x}{c}\right)}$$

$$= \frac{c\left(\cosh^2(x/c)\right)^{3/2}}{\cosh(x/c)}$$

$$= c\cosh^2(x/c)$$

Note:- length of normal

(4) Find the radius of curvature for $y =$

$y = ax^2 + bx + c$ at $x = \frac{1}{2a}\left(\sqrt{a^2-1} - b\right)$

$y_1 = 2ax + b$

$= \cancel{2a}\,\frac{1}{\cancel{2a}}\left(\sqrt{a^2-1} - b\right) + b$

$= \sqrt{a^2-1} - \cancel{b} + \cancel{b}$

$= \sqrt{a^2-1}$

$y_2 = 2a$

$$\rho = \frac{(1+y_1^2)^{3/2}}{y_2}$$

$$= \frac{1 + \left(\left(\sqrt{a^2-1}\right)^2\right)^{3/2}}{2a}$$

$a^2 - 1$

$$= \frac{1 + (a-1)^{3}}{2a}$$

$$= \frac{a^2}{2}$$

(5) Find the radius of curvature for the folium of decartes $x^3 + y^3 = 3axy$ at the point $\left(\frac{3a}{2}, \frac{3a}{2}\right)$ on it

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$$x^3 + y^3 = 3axy$$

$$3x^2 + 3y^2 \frac{dy}{dx} = 3a\left[x\frac{dy}{dx} + y\right]$$

$$3y^2 \frac{dy}{dx} - 3ax\frac{dy}{dx} = 3ay - 3x^2$$

$$3\frac{dy}{dx}\left(y^2 - ax\right) = 3\left(ay - x^2\right)$$

$$y_1 = \frac{ay - x^2}{y^2 - ax}$$

$$y_1 = \frac{a\left(\frac{3a}{2}\right) - \left(\frac{3a}{2}\right)^2}{\left(\frac{3a}{2}\right)^2 - a\left(\frac{3a}{2}\right)} = \frac{\frac{3a^2}{2} - \frac{9a^2}{4}}{\frac{9a^2}{4} - \frac{3a^2}{2}}$$

$$y_1 = \frac{\frac{3a^2}{2} - \frac{9a^2}{4}}{\frac{3a^2}{2}}$$

$$y_1 = \frac{\frac{6a^2 - 9a^2}{4}}{\frac{9a^2 - 6a^2}{4}} = \frac{-3\cancel{a^2}}{3\cancel{a}} = -1$$

$$y_2 = \frac{(y^2 - ax)(ay_1 - 2x) - (ay - x^2)(2yy_1 - a)}{(y^2 - ax)^2}$$

$$y_2 = \frac{\left(\left(3a/2\right)^2 - a\left(3a/2\right)\right)\left(a(-1) - 2\left(3a/2\right)\right) - \left(a\left(3a/2\right) - \left(3a/2\right)^2\right.}{\left(\left(\frac{3a}{2}\right)^2 - a\left(\frac{3a}{2}\right)\right)^2}$$

$$= \frac{\left(\frac{3a^2}{4}\right) - 4a - (-3a^3)(-4a)}{\left(\frac{3a^2}{4}\right)^2}$$

$$= \frac{-3a^3 - 3a^3}{\frac{9a^4}{16}}$$

$$= \frac{-6a^3 \times 16}{9a^4}$$

$$= \cancel{6a} \; \frac{-32}{3a}$$

$$\rho = \frac{(1 + y_1^2)^{3/2}}{y_2}$$

$$= \frac{(1 + (-1)^2)^{3/2}}{\frac{-32}{3a}}$$

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(6) Find the radius of curvature for the curve $y^2 = \frac{4a^2(2a-x)}{x}$ where the curve meets x-axis.

$$\frac{4a^2(2a-x)}{x} = 0$$

$$8a^3 - 4a^2x = 0$$

$$8a^{\cancel{3}} = 4\cancel{a^2}x$$

$$\cancel{8}^{2}a = \cancel{4}x$$

$$2a = x$$

$$\boxed{x = 2a}$$

$(2a, 0)$

$$y^2 = \frac{8a^3}{x} - 4a^2$$

$$2yy_1 = 8a^3\left(\frac{-1}{x^2}\right) - 0$$

$$y_1 = \frac{\frac{-\cancel{8}^{4}a^3}{x^2}}{\cancel{2}y} \Rightarrow y_1 = \frac{-4a^3}{x^2y}$$

$$y_1 = \frac{dy}{dx} = \frac{-4a^3}{x^2y}$$

$$(2a,0) \longrightarrow y_1 = \infty$$

$$x_1 = \frac{dx}{dy} = \frac{-x^2y}{4a^3}$$

$$(2a,0) = \boxed{x_1 = 0}$$

$$x_2 = \frac{-1}{4a^3}\left[x^2 \cdot 1 + y\,2x\right]$$

$(2a, 0) \rightarrow \boxed{x_2 = -\frac{1}{a}}$

$$\rho = \frac{(1+x_1^2)^{3/2}}{x_2} = \frac{(1+0)^{3/2}}{-1/a}$$

$$\rho = -a$$

$$|\rho| = a$$

(7) $x^2 y = a(x^2+y^2)$ $\quad (-2a, 2a)$

$$y \cdot 2x + x^2 \cdot y_1 = a(2x + 2y)$$

$$2xy + x^2 y_1 = 2ax + 2ay$$

$$x^2 y_1 = \frac{2ax + 2ay}{2xy}$$

$$x^2 y_1 = \frac{\cancel{2}(ax+ay)}{\cancel{2}xy}$$

$$y \cdot 2x + x^2 y_1 = a(x^2+y^2)$$

$$= a(2x + 2y \cdot y_1)$$

$$= 2ax + 2ayy_1$$

$$x^2 y_1 - 2ayy_1 = 2xy + 2ax$$

$$y_1 = \frac{2a(xy + ax)}{(x^2 - 2ay)}$$

$$= \frac{2(-4a^2 - (-2a^2))}{4a^2 - 2(2a^2)}$$

$$= \frac{1}{0} = \infty$$

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$$x_1 = \frac{dx}{dy} = \frac{x^2 - 2ay}{2ax - 2xy} \quad \frac{U}{V}$$

$$x_1 = \frac{dx}{dy} = \frac{1}{y_1} = \frac{x^2 - 2ay}{2ax - 2xy} \text{ at } (-2a, 2a)$$

$$(x_1 = 0)$$

$$x_2 = \frac{d^2x}{dy^2} = \frac{(2ax - 2xy)(2x x_1 - 2a) - (x^2 - 2ay)(2ax_1 - 2x - 2x_1 y)}{(2ax - 2xy)^2}$$

Point $(-2a, 2a)$

$(2ax - 2xy) = 4a^2$ & $(x^2 - 2ay) = 0$

$$\therefore (x_2)_{(-2a, 2a)} = \frac{(4a^2)(-2a)}{16a^4} = -\frac{1}{2a}$$

$$\rho = \frac{(1 + x_1^2)^{3/2}}{x_2} = \frac{(1)^{3/2}}{-1/2a} = -2a$$

$$|\rho| = 2a$$

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(8) S.T the radius of curvature at any point θ on the cycloid $x = a(\theta + \sin\theta)$ $y = a(1 - \cos\theta)$ is $4a \cos\frac{\theta}{2}$

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Sol:- $x = a(\theta + \sin\theta)$ $\qquad y = a(1 - \cos\theta)$

$\frac{dx}{d\theta} = a(1 + \cos\theta)$

$y_1 = \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$

$= \frac{\cancel{a}\sin\theta}{\cancel{a}(1+\cos\theta)}$

$= \frac{\cancel{2}\sin\theta/2 \cdot \cancel{\cos\theta/2}}{\cancel{2}\cos^{\cancel{2}}\theta/2}$

$y_1 = \tan(\theta/2)$

$y_2 = \sec^2(\theta/2) \cdot \frac{1}{2}\frac{d\theta}{dx}$ $\qquad = \sec^2\theta/2 \cdot \frac{1}{2}\,\frac{1}{a(1+\cos\theta)}$

$\rho = \frac{(1+y_1^2)^{3/2}}{y_2}$

$\rho = \frac{(1+\tan^2\theta/2)^{3/2}}{1/2\,\sec^2(\theta/2)}$ $\qquad = \sec^2\theta/2 \cdot \frac{1}{2} \cdot \frac{1}{a(2\cos^2\theta/2)}$

$= \frac{2(\sec^2\theta/2)^{3/2}}{\sec^2\theta/2}$ $\qquad = \frac{1}{4a}\sec^2\theta/2 \cdot \sec^2\theta/2 = \frac{1}{4a}\sec^4\theta/2$

$\rho = \frac{(1+\tan^2\theta/2)^{3/2}}{\frac{1}{4a}\sec^4\theta/2} = \frac{4a(\sec^2\theta/2)^{3/2}}{\sec^4\theta/2}$

$= \frac{4a\sec^3\theta/2}{\sec^4\theta/2} = 4a\,\frac{1}{\sec\theta/2}$

$= 4a\cos\theta/2$

$y_1 = -\tan\theta$
$y_2 = \sec^4\theta$

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(2) $x = a\cos^3\theta$ $\qquad y = a\sin^3\theta$

$\frac{dx}{d\theta} = a\,(\cos^3\theta)$

$y_1 = \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$

$= \frac{dx}{d\theta} = a\,3\cos^2\theta . \sin\theta$

$\frac{dy}{d\theta} = a\,3\sin^2\theta . \cos\theta$

$y_1 = \frac{dy}{dx} = \frac{3\cancel{a}\sin^2\theta . \cancel{\cos\theta}}{\cancel{a}-3\cancel{a}\cos^2\theta . \sin\theta}$

$y_1 = -\tan\theta$

$y_2 = -\sec^2\theta = \frac{d\theta}{dx}$

$= \cancel{+}\sec^2\theta . \frac{1}{\cancel{+}3a\cos^2\theta . \sin\theta}$

$= \frac{\sec^4\theta}{3a.\sin\theta}$

$\rho = \frac{(1+y_1^2)^{3/2}}{y_2}$

$\rho = \frac{(1+(-\tan\theta))^{3/2}}{\frac{\sec^4\theta}{3a\sin\theta}}$

$= \frac{(1+\tan^2\theta)^{3/2}}{} = \frac{(\sec^2\theta)^{3/2}}{\sec^4\theta}$

$= \frac{3a\sin\theta}{\sec\theta} = 3a\sin\theta . \cos\theta$

$3a . \frac{1}{\sqrt{2}} . \frac{1}{\sqrt{2}} = \cancel{3a}$ ~~3a × 1.414~~

$= 3a/2$

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Radius of curvature of polar form

$$\boxed{\rho = \frac{(r^2 + r_1^2)^{3/2}}{(r^2 + 2r_1^2 - r r_2)}}$$

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1. Find the radius of curvature for $r = ae^{\theta \cot\alpha}$

$\log r = \log a + \theta \cot\alpha \log e$

$\frac{1}{r}\frac{dr}{d\theta} = 0 + \theta \cot\alpha \cdot 1$

$r_1 = r\theta \cot\alpha$

$r_2 = \cot\alpha \, r_1$

$= \cot\alpha \, (r\cot\alpha)$

$= r\cot^2\alpha$

$$\rho = \frac{(r^2 + r_1^2)^{3/2}}{(r^2 + 2r_1^2 - r r_2)}$$

$$\rho = \frac{(r^2 + r^2\cot^2\alpha)^{3/2}}{r^2 + 2r^2\cot^2\alpha - r(r\cot^2\alpha)}$$

$$= \frac{(r^2)^{3/2}(1+\cot^2\alpha)^{3/2}}{(r^2 + r^2\cot^2\alpha)}$$

$$= \frac{r^3(\mathrm{cosec}^2\alpha)^{3/2}}{r^2(1+\cot^2\alpha)}$$

$$= \frac{\cancel{r^3}\,\mathrm{cosec}^3\alpha}{\cancel{r^2}\,\cancel{\mathrm{cosec}^2\alpha}}$$

$$\boxed{\rho = r\,\mathrm{cosec}\,\alpha}$$

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2. $r^n = a^n \cos n\theta$

$n \log r = n \log a + \log \cos n\theta$

$n \cdot \frac{1}{r} \frac{dr}{d\theta} = \cancel{n \log a} + 0 + \frac{1}{\cos n\theta} (-\sin n\theta) \cdot n$

$\cancel{n} \times r' = \frac{r - \sin n\theta \cancel{n}}{\cos n\theta}$

$r_1 = -r \tan\theta$

$r_2 = -r (\sec^2 n\theta) n + \tan\theta \frac{dr}{d\theta}$

$= -rn \sec^2 n\theta - \tan\theta \times r_1$

$\rho = \frac{(r^2 + r_1^2)^{3/2}}{(r^2 + 2r_1^2 - rr_2)}$

$= \frac{(r^2 + (-r \tan n\theta)^2)^{3/2}}{r^2 + 2(-r \tan\theta)^2 - r(-rn \sec^2\theta - r_1 \tan n\theta)}$

$= \frac{(r^2 + r^2 \tan^2 n\theta)^{3/2}}{r^2 + 2r^2 \tan^2 n\theta + r^2 n \sec^2 n\theta + 2r_1 \tan\theta}$

$= \frac{(r^2)^{3/2} (1 + \tan^2\theta)^{3/2}}{r^2 + 2r^2 \tan^2 n\theta + r^2 n \sec^2 n\theta - r^2 \tan^2 n\theta}$

$= \frac{r^3 (\sec^2 n\theta)^{3/2}}{r^2 + r^2 \tan^2 n\theta + nr^2 \sec^2 n\theta}$

$= \frac{\cancel{r^3} \sec^3 n\theta}{\cancel{r^2} (1 + \tan^2 n\theta + n \sec^2 n\theta)}$

$$\frac{r\sec^3 n\theta}{\sec^2 n\theta + n\sec^2 n\theta}$$

$$= \frac{r\sec^3 n\theta}{\sec^2 n\theta(1+n)} = \frac{r\sec n\theta}{1+n}$$

$$\rho = \frac{r}{1+n}\left(\frac{a^n}{r^n}\right) = \frac{a^n}{1+n}\left(r^{n-1}\right)$$

3. $r = a(1+\cos\theta)$

$$\log r = \log a + \log(1+\cos\theta)$$

$$\frac{1}{r}\cdot\frac{dr}{d\theta} = 0 + \frac{1}{1+\cos\theta}(-\sin\theta)$$

$$r_1 = r\left(\frac{-\cancel{2}\sin\theta/2 \cdot \cos\theta/2}{\cancel{2}\cos^2\theta/2}\right)$$

$$r_1 = -r\tan\theta/2$$

$$r_2 = -r\sec^2\theta/2 - \tan\theta/2\left(\frac{dr}{d\theta}\right)\left(\frac{1}{2}\right)$$

$$r_2 = -\frac{1}{2}r\sec^2\theta/2 - r_1\tan\theta/2$$

$$= -\frac{1}{2}r\sec^2\theta/2 + r\tan^2\theta/2$$

$$\rho = \frac{\left(r^2+r_1^2\right)^{3/2}}{\left(r^2+2r_1^2-rr_2\right)}$$

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$$= \frac{(r^2 + r^2\tan^2\theta/2)^{3/2}}{r^2 + 2r^2\tan^2\theta/2 - r\left(-\frac{1}{2} r\sec^2\theta/2 + r\tan^2\theta/2\right)}$$

$$= \frac{(r^2)^{3/2}(1+\tan^2\theta)^{3/2}}{r^2 + 2r^2\tan^2\theta/2 - r^2\tan^2\theta/2 + 1/2\, r\sec^2\theta}$$

$$= \frac{r^3\sec^3\theta/2}{r^2 + r^2\tan^2\theta/2 + 1/2\, r^2\sec^2\theta/2}$$

$$= \frac{r^{\cancel{3}}\sec^{\cancel{3}}\theta/2}{\cancel{r^2}\cancel{\sec^2\theta/2}\left(1+\frac{1}{2}\right)}$$

$$= \frac{r\sec\theta/2}{(1+1/2)} = \frac{2}{3} r\sec\theta/2$$

$$\boxed{\rho = 2/3\, r\sec\theta/2}$$

Centre of Curvature

$$\boxed{\bar{x} = x - \frac{y_1(1+y_1^2)}{y_2}\ ;\ \bar{y} = y + \frac{1+y_1^2}{y_2}}$$

Circle of Curvature

2/11/18 $(x-\bar{x})^2 + (y-\bar{y})^2 = \rho^2$

1) $y = x + \frac{9}{x}$ at $(3,6)$

$$y_1 = 1 + 9\left(\frac{-1}{x^2}\right)$$

$$y_2 = 0 - 9(-2x^{-3})$$

$$= \frac{18}{x^3} \quad \text{At } (3,6)\ \boxed{y_2 = 2/3}$$

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$$\bar{x} = 3 - \frac{0}{2/3}[1+0]$$

$$\boxed{\bar{x} = 3}$$

$$\bar{y} = 6 + \left[\frac{1+0}{2/3}\right] \Rightarrow \boxed{\bar{y} = \frac{5}{2}}$$

$$\rho = \frac{(1+y_1^2)^{3/2}}{3/2} \Rightarrow \boxed{\rho = \frac{3}{2}}$$

$$(x-\bar{x})^2 + (y-\bar{y})^2 = \rho^2$$

$$(x-3)^2 + (y - 15/2)^2 = \frac{9}{4}$$

2) $y^2 = 12x$ at $(3,6)$

$$2yy_1 = 12$$

$$y_1 = \frac{6}{y} = \frac{6}{6} = 1$$

$$y_2 = \frac{-6}{y_1^2} \cdot y_1$$

$$\boxed{y_2 = -6}$$

$$y_2 = \frac{-\cancel{6}}{\cancel{36}_{6}} = \frac{-1}{6}$$

$$\boxed{y_2 = -1/6}$$

$$\bar{x} = 3 - \frac{1}{-1/6}(1+1)$$

$$\bar{x} = 3 - 6(1+1)$$

$$\bar{x} = 3 + 12$$

$$\bar{x} = 15$$

$$\bar{y} = 6 + \left(\frac{1+1}{-1/6}\right)$$

$$\bar{y} = 6 - (12)$$

$$\bar{y} = -6$$

$$\rho = \frac{(1+y_1^2)^{3/2}}{3/2} = \rho = \frac{2^{3/2}}{-1/6}$$

$$= -\frac{(12)^{3/2}}{1/6}$$

$$\frac{(2^3)^{3/2}}{-1}$$

$$(x-15)^2 + (y+6)^2 = \cancel{16}$$

$$(x-15)^2 + (y+6)^2 = 288 \qquad \rho = 4$$

Sl. No	Name of the curve	Cartesian form	Parametric form
1.	(i) Parabola (ii) Parabola	$y^2 = 4ax$ $x^2 = 4ay$	$x = at^2, y = 2at$ $x = 2at, y = at^2$
2.	Ellipse	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	$x = a\cos t$, $y = b\sin t$
3.	Hyperbola	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$x = a\sec t$, $y = b\tan t$
4.	Rectangular Hyperbola	$xy = c^2$	$x = ct, y = c/t$
5.	Astroid	$x^{2/3} + y^{2/3} = a^{2/3}$	$x = a\cos^3 t$, $y = a\sin^3 t$

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Find the evolute & involutes

Find the evolute of Parabola

(1) $y^2 = 4ax$

$x = at^2$ $\qquad$ $y = 2at$

$\frac{dx}{dt} = 2at$ $\qquad$ $\frac{dy}{dt} = 2a$

$$y_1 = \frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

$$= \frac{2a}{2at} \Rightarrow \boxed{y_1 = \frac{1}{t}}$$

$$y_2 = -\frac{1}{t^2}\frac{dt}{dx}$$

$$= -\frac{1}{t^2}\frac{1}{2at}$$

$$\boxed{y_2 = \frac{-1}{2at^3}}$$

$$\bar{x} = x - \frac{y_1}{y_2}\left[1 + y_1^2\right]$$

$$= at^2 - \frac{1/\cancel{t}}{-1/2at^{\cancel{3}2}}\left[1 + \frac{1}{t^2}\right]$$

$$= at^2 + 2at^2\left[1 + \frac{1}{t^2}\right]$$

$$= at^2 + 2at^2 + 2a$$

$$\boxed{\bar{x} = 3at^2 + 2a}$$

$$\bar{y} = y + \left[\frac{1 + y_1^2}{y_2}\right]$$

$$= 2at + \left[\frac{1 + 1/t^2}{-1/2at^3}\right]$$

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$$= 2at - 2at^3\left[1+\frac{1}{t^2}\right]$$

$$= \cancel{2at} - 2at^3 - \cancel{2at}$$

$$\boxed{\bar{y} = -2at^3}$$

Consider, $\bar{x} = 3at^2 + 2a$

$$t^2 = \frac{\bar{x}-2a}{3a}$$

$$(t^2)^{3/2} = \left(\frac{\bar{x}-2a}{3a}\right)^{3/2}$$

$$\bar{y} = \frac{-2a\,(\bar{x}-2a)^{3/2}}{(3a)^{3/2}}$$

Squa on b.s

$$(\bar{y})^2 = 4a^2\frac{(\bar{x}-2a)^3}{(3a)^3}$$

$$= \frac{4\cancel{a^2}\,(\bar{x}-2a)^3}{27\,a^{\cancel{3}}}$$

$$= 27a(\bar{y})^2 = 4(\bar{x}-2a)^3$$

$$27ay^2 = 4(x-2a)^3$$

(2) Find the evolute of Ellipse

$x = a\cos t$ $\qquad$ $y = b\sin t$

$\frac{dx}{dt} = -a\sin t$ $\qquad$ $\frac{dy}{dt} = b\cos t$

$$y_1 = \frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

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$$= \frac{b\cos t}{-a\sin t} \qquad \boxed{y_1 = \frac{-b}{a}\cot t}$$

$$y_2 = \frac{-b}{a}(-\operatorname{cosec}^2 t)\frac{dt}{dx}$$

$$= \frac{b}{a}\operatorname{cosec}^2 t \, \frac{1}{-a\sin t}$$

$$\boxed{y_2 = \frac{-b}{a^2}\operatorname{cosec}^3 t}$$

$$\bar{x} = x - \frac{y_1}{y_2}(1+y_1^2)$$

$$= a\cos t - \frac{-\cancel{b/a}\cot t}{-\cancel{b/a}\operatorname{cosec}^3 t}\left[1 + \frac{b^2\cos^2 t}{a^2}\right]$$

$$= a\cos t - a\sin^{\cancel{3}2} t\,\frac{\cos t}{\cancel{\sin t}}\left[1 + \frac{b^2}{a^2}\cot^2 t\right]$$

$$= a\cos t - a\sin^2 t\cos t - a\cancel{\sin^2 t}\cos t\,\frac{b^2}{\cancel{a^2}}\,\frac{\cos^2 t}{\cancel{\sin^2 t}}$$

$$= a\cos t - a\sin^2 t\cos t - \frac{b^2}{a}\cos^3 t$$

$$= a\cos t(1-\sin^2 t) - \frac{b^2}{a}\cos^3 t$$

$$= a\cos^3 t - \frac{b^2}{a}\cos^3 t$$

$$= \cos^3 t\left(a - \frac{b^2}{a}\right)$$

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$$\boxed{\bar{x} = \left(\frac{a^2-b^2}{a}\right)\cos^3 t}$$

$$\bar{y} = y + \left(\frac{1+y_1^2}{y_2}\right)$$

$$= b\sin t + \left[\frac{1 + b^2/a^2 \cot^2 t}{-b/a^2 \operatorname{cosec}^3 t}\right]$$

$$= b\sin t - \frac{a^2 \sin^3 t}{b}\left[1 + \frac{b^2}{a^2}\frac{\cos^2 t}{\sin^2 t}\right]$$

$$= b\sin t - \frac{a^2}{b}\sin^3 t - \frac{\cancel{a^2} b^{\cancel{2}}}{\cancel{b}\cancel{a^2}}\cancel{\sin^3 t}\frac{\cos^2 t}{\cancel{\sin^2 t}}$$

$$= b\sin t - \frac{a^2}{b}\sin^3 t - b\sin t\cos^2 t$$

$$= b\sin t - \frac{a^2}{b}\sin^3 t - b\sin t\left(1-\sin^2 t\right)$$

$$= \cancel{b\sin t} - a^2/b \sin^3 t - \cancel{b\sin t} + b\sin^3 t$$

$$\boxed{\bar{y} = -\left(\frac{a^2-b^2}{b}\right)\sin^3 t}$$

$$(\bar{x})^{2/3} = \left(\frac{a^2-b^2}{a}\right)^{2/3}\left(\cos^3 t\right)^{2/3}$$

$$(a\bar{x})^{2/3} = (a^2-b^2)^{2/3}\cos^2 t \rightarrow (1)$$

$$(\bar{y})^{2/3} = -\left(\frac{a^2-b^2}{b}\right)^{2/3}\left(\sin^3 t\right)^{2/3}$$

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$(b\bar{y})^{2/3} = (a^2-b^2)^{2/3} \sin^2 t \rightarrow (2)$

(1) + (2)

$= (a\bar{x})^{2/3} + (b\bar{y})^{2/3} = (a^2-b^2)^{2/3} \cos^2 t + (a^2-b^2)^{2/3} \sin^2 t$

$= (a^2-b^2)^{2/3} (\cos^2 t + \sin^2 t) \quad (ax)^{2/3} + (by)^{2/3} = (a^2-b^2)^{2/3}$

(3) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ (Hyperbola)

$+\frac{b}{a} \operatorname{cosec} t$

$x = a\sec t \qquad y = b\tan t$

$\frac{dx}{dt} = a\sec t \tan t \qquad \frac{dy}{dt} = b\sec^2 t$

$y_1 = \frac{dy}{dx} = \frac{dy/dt}{dx/dt}$

$= \frac{b\sec^2 t}{a\sec t \tan t}$

$\boxed{y_1 = \frac{b}{a} \operatorname{cosec} t}$

$y_2 = -\frac{b}{a} (-\cot^2 t) \frac{dt}{dx}$

$= \frac{b}{a} \cot^2 t \, \frac{1}{a\tan t}$ cot t

$\boxed{y_2 = -\frac{b}{a^2} \cot^3 t}$

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$$\bar{x} = x - \frac{y_1}{y_2}(1+y_1^2)$$

$$= a\sec t - \frac{\frac{b}{a}\operatorname{cosec}t}{-\frac{b}{a^2}\cot^3 t}\left(1+\frac{b}{a}\operatorname{cosec}t\right)$$

$$= a\sec t - \frac{-a^2}{b} \times \frac{\frac{b}{a}\operatorname{cosec}t}{\cot^3 t}\left(1+ b/a \operatorname{cosec} t\right)$$

$$= a\sec t + a\frac{\operatorname{cosec}t}{\cot^3 t}\left(1+ b^2/a^2 \operatorname{cosec}t\right)$$

$$= a\sec t + a\frac{1}{\sin t}\frac{\sin^3 t}{\cos^3 t}\left(1+\frac{b^2}{a^2}\operatorname{cosec}^2 t\right)$$

$$= a\sec t + a\frac{\sin^2 t}{\cos^3 t} + \frac{\cancel{a}b^2}{a^{\cancel{2}}}\frac{\sin^2 t}{\cos^3 t}\frac{1}{\sin^2 t}$$

$$= a\sec t + a\frac{\sin^2 t}{\cos^3 t} + \frac{b^2}{a}\sec^3 t$$

$$= a\sec t + \frac{a(1-\cos^2 t)}{\cos^3 t} + \frac{b^2}{a}\sec^3 t.$$

$$= \cancel{a\sec t} + a\sec^3 t - \cancel{a\sec t} + \frac{b^2}{a}\sec^3 t$$

$$\bar{x} = \left(\frac{a^2+b^2}{a}\right)\sec^3 t$$

$$\bar{y} = y + \frac{(1+y_1^2)}{y_2}$$

$$= b\tan t + \frac{\left(1+\frac{b^2}{a^2}\operatorname{cosec}^2 t\right)}{-b/a^2 \cot^3 t}$$

$$= b\tan t + \frac{\left(-\frac{a^2}{b}\times\frac{b^2}{a^2}\operatorname{cosec}^2 t\right)}{\cot^3 t}$$

$$= b\tan t - \frac{b\operatorname{cosec}^3 t}{\cot^3 t}$$

$$= b\tan t - \frac{a^2}{b}\cot^3 t\left[1+\frac{b^2}{a^2}\operatorname{cosec}^3 t\right]$$

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$= b\tan t - \frac{a^2}{b} \frac{\sin^3 t}{\cos^3 t}\left[1 + \frac{b^2}{a^2}\operatorname{cosec}^3 t\right]$

$= b\tan t - \left[\frac{a^2}{b}\frac{\sin^3 t}{\cos^3 t}\left\{-\frac{1}{a}\frac{\sin^3 t}{\cos^3 t}\cdot\frac{1}{\cancel{\sin^3 t}}\right]\right.$

$= b\tan t - \frac{a^2}{b^2}\tan^3 t - \cancel{\frac{b(1-\cos t)}{\cos^3 t}} \cdot \frac{b\sin t}{\cos^3 t}$

$=$

$= b\tan t - \frac{a^2}{b}\tan^3 t - \cancel{b} - b\tan t\cdot\sec^2 t$

$= b\tan t - \frac{a^2}{b}\tan^3 t - b\tan t\,(1+\tan^2 t)$

$= b\cancel{\tan t} - \frac{a^2}{b}\tan^3 t - \cancel{b\tan t} - b\tan^3 t$

$= - a^2/b\,\tan^3 t - b\tan^3 t$

$\boxed{\bar{y} = -(a^2/b + b)\tan^3 t.}$

$\boxed{\bar{y} = -\left(\frac{a^2+b^2}{b}\right)\tan^3 t}$