Module - 2.

Single phase circuits.

1. How to draw a phasor diagram :-

In a ac circuit the voltages and currents are phasors, so the voltages must be added treating them as phasor.

Step 1 :- Mark the source voltage V, showing its polarity, either by an arrow or by using + and - signs. Mark the source current I showing its direction by an arrow. As a convention, the current I must leave the positive terminal of the source.

Step 2 :- Mark 'the voltage across' and 'the current through' each individual components of the circuit, following the passive sign convention. We have marked V_R and I_R for the resistance R and V_L and I_L for the inductance L.

~~Step 3 :- To get complete phasor diagram, super impose all the individual phasor diagram, by recogonising the common phasor among them.~~

Step 3 :-

Draw the phasor diagram for individual components.

(i) For resistance, R: The current is in phase with the voltage. Draw the voltage phasor V_R along the reference direction. Draw the current phasor I_R also along the reference direction.

(ii) For inductance, L :- The current lags the voltage by 90°. Draw the voltage phasor V_L along the reference direction. Draw the current phasor I_L 90° lagging

Step 4 :- To get complete phasor diagram, Superimpose all the individual phasor diagram by recognising the common phasor among them.

Step 5:-

Find the phasor addition of V_R and V_L by drawing the parallelogram OABC

Step 6 :-

Once the phasor diagram is drawn, we can take the help of complex algebra to make calculations marking the reference direction as positive real axis and the imaginary axis to be y.

$I = I\angle 0^\circ \quad V_R = IR \quad V_L = jIX_L = IjX_L = Ij\omega L$

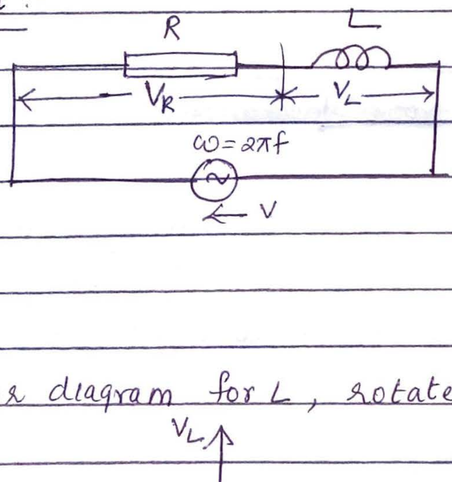
$V = V_R + V_L$

$= IR + \cancel{IX_L}$

$= I_1 R + IX_L$

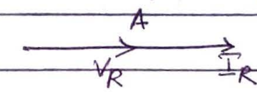
$= I(R + X_L).$

circuit :-

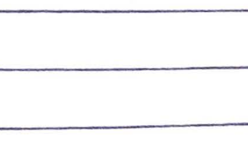


Phasor diagram of 'R'

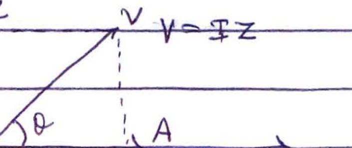
(a)



(b) Phasor diagram of L



(c) Phasor diagram for L, rotated by 90°



(d) The complete phasor diagram

(2) What is impedence triangle.

* For an ac circuit, the ratio of the Voltage phasor to the current phasor is a complex quantity, called complex impedance.

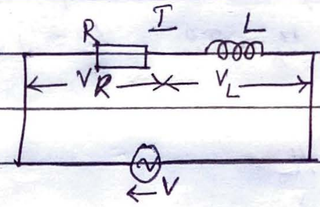
* The real part is called resistance and its imaginary part is called reactance. Thus,

complex impedance = (resistance) + j (reactance)

$$Z = R + jX$$

For the series RL circuit,

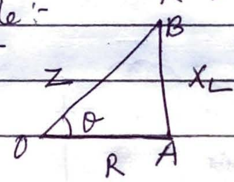
$$Z = \frac{V}{I} = R + j\omega L = Z\angle\theta$$



$$Z = \sqrt{R^2 + (\omega L)^2}$$

$$\theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

Impedence triangle :-



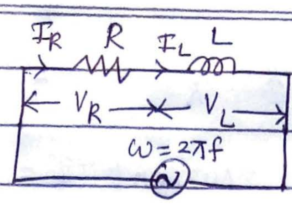
From the phasor diagram, we can seperate the voltage triangle OAB. If each side of this triangle is divided by I, the result is the impedence triangle.

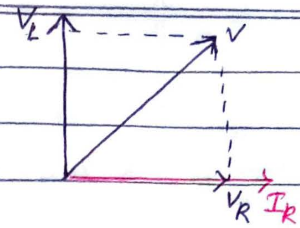
(3) Series RL and RC Circuit :-

Series RL circuit

* The reference polarity is given by V.
* The V_R and V_L be the voltage drops across resistance R and inductance L, respectively.
* $V = V_R + V_L$

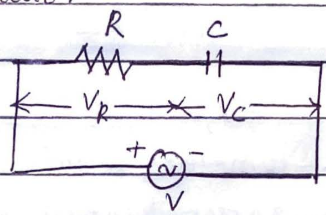
* circuit

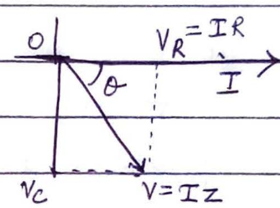




* $V = IR + IX_L$
$= I_R + Ij\omega L$
$= I(R + j\omega L)$
$V = I(R + X_L)$.

Series RC circuit:-





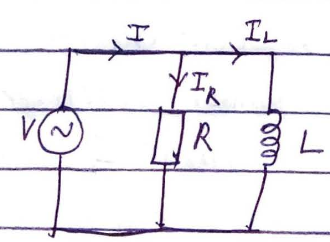
* V_R and V_C be the voltages drops across resistance R and enductance L, respectively.

* $V = V_R + V_C$
$= IR + IX_C$
$= I(R + 1/j\omega C)$
$= I(R + X_C)$

* Take current phasor as reference. V_R is in phase with the current I, but for the capacitance, the voltage V_C is drawn lagging the current I by 90°.

* $V = I(R + X_C)$

(4) Parallel RL and RC circuit :-



* The current through the resistance is I_R and the current through the inductance is I_L

$$I_R = \frac{V}{R} \qquad I_L = \frac{V}{jX_L}$$

* The total current $I = I_R + I_L$

$$I = \frac{V}{R} + \frac{V}{X_L}$$

$$= V\left(\frac{1}{R} + \frac{1}{X_L}\right)$$

$$Y = \frac{1}{R} + \frac{1}{X_L}$$

* The current, $I = VY$
where Y is the complex admittance of the parallel RL circuit. After rationalisation, we can write

$$Y = \frac{1}{R} + \frac{1}{X_L} = \frac{1}{R} + \frac{1}{j\omega L}$$

$$= \frac{1}{R} - j\frac{1}{\omega L}$$

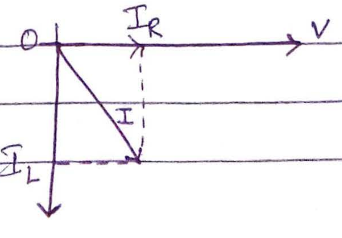
* In general, the real part of complex admittance Y is called conductance and imaginary part is called susceptance. Thus, we can write

$$Y = G + jB$$

$$= \text{conductance} + j\ \text{susceptance}$$

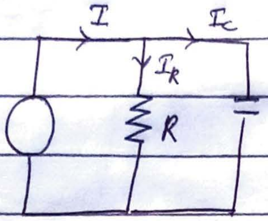
* $I = I_R + I_L = VY = V(G + jB)$
* phasor diagram :-

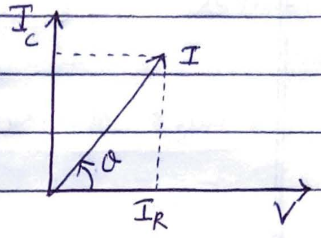
We start the phasor diagram by taking the voltage phasor as reference. The current I_R is in phase with V and current I_L lags voltage V by $90°$. The resultant current phasor I is then found by adding phasors I_R and I_L.



Parallel RC circuit :-

The current I_R through resistor R and the current I_c through capacitor C are given as,





$$I_R = \frac{V}{R}$$

$$I_c = \frac{V}{-jX_c} = \frac{V}{-j(1/\omega C)} = Vj\omega C$$

using KCL, the total current

$$I = I_R + I_c$$

$$= \frac{V}{R} + V(j\omega C).$$

$$= V\left(\frac{1}{R} + j\omega C\right)$$

$$= V(Y).$$

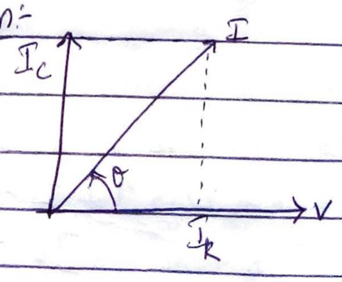
where $Y = \frac{1}{R} + j\omega C = G + jB.$

Conductance $G = 1/R$.

Susceptance $B = \omega C$

$$\theta = \tan^{-1}\left(\frac{B}{G}\right) = \tan^{-1}\left(\frac{\omega C}{1/R}\right)$$

Phasor diagram:-

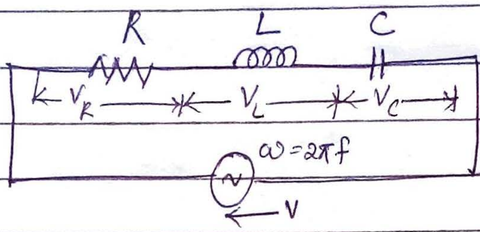


* Taking voltage V as reference, the current I_R is in phase with voltage V, but the current I_c leads the

Voltage by 90°. The result current phasor I is then found by adding phasors I_R and I_C.

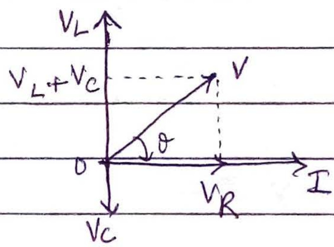
(5) Series RLC circuit :-

circuit diagram :-

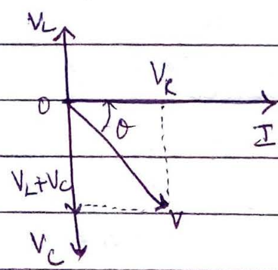


phasor diagram :-

when $\omega L > 1/\omega C$ (or) $X_L > X_C$



when $\omega L < 1/\omega C$ (or) $X_L < X_C$



KVL equation :-

$$V = V_R + V_L + V_C$$
$$= IR + I(jX_L) + I(-jX_C)$$
$$= I[R + jX_L - jX_C] = IZ$$

where Z is the complex impedance of the given circuit

$$Z = R + j(X_L - X_C)$$
$$= R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \angle \tan^{-1} \frac{\omega L - (1/\omega C)}{R}$$

$$Z = |Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$\theta = \tan^{-1} \frac{\omega L - (1/\omega C)}{R}$$

$$I = I\angle\theta = \frac{V}{Z} = \frac{V\angle 0^\circ}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} \angle -\tan^{-1} \frac{\omega L - (1/\omega C)}{R}$$

RLC Circuit

Inductive circuit:-

when $\omega L > 1/\omega C$ the phase angle ϕ of the current phasor is negative. The current lags the voltage. The circuit behaves as an inductive circuit

Capacitive circuit:-

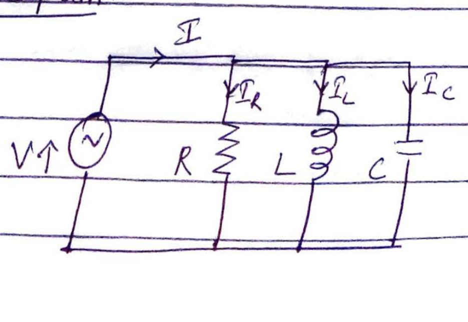
When $\omega L > 1/\omega C$ the phase angle ϕ of the current phasor is positive. The current leads the voltage. The circuit behaves as a capacitive circuit.

purely resistive circuit:-

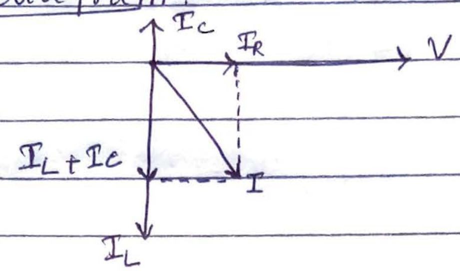
When $\omega L = 1/\omega C$ the phase angle $\phi = 0$. The current is in phase with voltage. The circuit behaves as a purely resistive circuit. This is special case and is called resonance.

6 Parallel RL Circuit:-

circuit diagram:-



Phasor diagram :-



KCL equation :-

$$I = I_R + I_L + I_C$$

$$= \frac{V}{R} + \frac{V}{jX_L} + \frac{V}{-jX_C}$$

$$= VG + V(-jY_L) + V(jY_C)$$

$$I = V[G + j(Y_C - Y_L)] = VY$$

Y is the complex admittance of the given circuit

$$G = \frac{1}{R} \; ; \; Y_L = \frac{1}{X_L} = \frac{1}{\omega L} \; ; \; Y_C = \frac{1}{X_C} = \frac{\omega C}{1} = \omega C$$

$$Y = G + j(Y_C - Y_L) = \sqrt{G^2 + (Y_C - Y_L)^2} \tan^{-1} \frac{Y_C - Y_L}{G}$$

The current equation :-

$$I = I\angle\phi = VY = V\sqrt{G^2 + (Y_C - Y_L)^2} \angle \tan^{-1} \frac{Y_C - Y_L}{G}$$

Problems :-

1. For the series RL circuit.

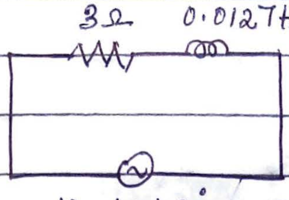
(a) calculate the rms value of the steady state current and relative phase angle.

(b) Write the expression for the instataneous current.

(c) Find the average power dissipated in the circuit.

(d) Determine the power factor.

(e) Draw the phasor diagram.



$V = 141 \sin 100\pi t$ V

$V = V\angle 0^\circ = \frac{V_m}{\sqrt{2}} \angle 0^\circ = \frac{141}{\sqrt{2}} \angle 0^\circ = 100\angle 0^\circ = 100 + j0$ V.

$Z = R + j\omega L$

$= 3 + j\, 100\pi \times 0.0127 = 3 + j4 = 5\,\angle 53.1^\circ\, \Omega$.

current $I = \frac{V}{Z} = \frac{100\angle 0^\circ}{5\angle 53.1} = 20\angle -53.1^\circ$ A

The rms value of the steady-state current is 20A, the phase angle is 53.1° lagging.

(b) The instantaneous current can be written as,

$i = 20\sqrt{2} \sin(100\pi t - 53.1^\circ)$ A.

$= 28.28 \sin(100\pi t - 53.1^\circ)$ A.

(c) average power $P = VI\cos\theta$

$= 100 \times 20 \times \cos 53.1^\circ = 1200$ W.

$P = I^2 R$

$= (20)^2\, 3 = 1200$ W.

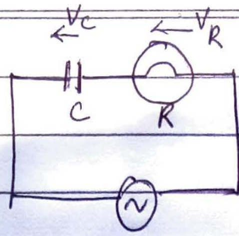
(d) $pf = \cos\theta = \cos 53.1^\circ = 0.6$ lagging

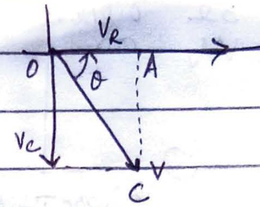
(e) $V_R = IR = 20 \times 3 = 60$ V

$V_L = IX_L = 20 \times 4 = 80$ V

$V = 100$ V.

2. A metal filament lamp, rated at 750W, 100V, is to be used on a 230V, 50Hz supply by connecting a capacitor of suitable value in series. Determine (a) the capacitance required (b) the phase angle (c) the power factor (d) the apparent power (e) the reactive power.





$$I = \frac{P}{V_R} = \frac{750}{100}$$

$$I = 7.5\,A$$

(a) From ΔOAC we can determine the voltage across the capacitor.

$$V_c = \sqrt{V^2 - V_R^2} = \sqrt{(230)^2 - (100)^2} = 207\,V$$

$$X_C = \frac{V_c}{I} = \frac{207}{7.5} = 27.6\,\Omega \quad \text{or} \quad \frac{1}{2\pi f C} = 27.6$$

$$C = \frac{1}{2\pi \times 50 \times 27.6} = 115 \times 10^{-6}\,F = 115\,\mu F.$$

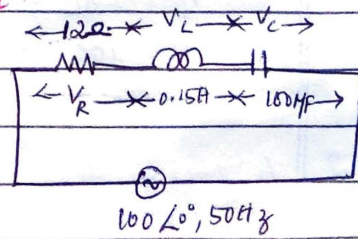
(b) phase angle, $\phi = \cos^{-1}\frac{V_R}{V} = \cos^{-1}\frac{100}{230} = 64^\circ 12'$

(c) power factor $\cos\phi = (100/230) = 0.435$ ~~lagging~~ leading.

(d) Apparent power $= VI = 230 \times 7.5 = 1725\,VA$

(e) Reactive power $= VI\sin\phi = 230 \times 7.5 \times \sin 64^\circ 12' = 1553\,VAR$.

3. For the circuit shown calculate (a) the impedance (b) the current (c) the phase angle (d) the voltage across each element (e) the power factor (f) the apparent power (g) the average power. Also draw the phasor diagram for the circuit.



$$X_L = \omega L = 2\pi f L = 100\pi \times 0.15$$
$$= 47.1\,\Omega$$

$$X_c = \frac{1}{\omega C} = \frac{1}{(100\pi \times 100 \times 10^{-6})} = 31.8\,\Omega$$

(a) The impedence $Z = R + j(X_L - X_C)$

$$= 12 + j(47.1 - 31.8)$$
$$= (12 + j15.3)\,\Omega$$
$$= \sqrt{12^2 + 15.3^2}\,\angle\tan^{-1}(15.3/12)$$
$$= 19.4\,\angle 51.9^\circ\,\Omega.$$

(b) The current, $I = \frac{V}{Z} = \frac{100 \angle 0^\circ}{19.4 \angle 51.9^\circ} = 5.15 \angle -51.9^\circ A$.

(c) The phase angle $\phi = -51.9^\circ$

(d) The voltage, $V_R = IR = 5.15 \times 12 = 61.8 V$

$V_L = IX_L = 5.15 \times 47.1 = 242.6 V$

$V_C = IX_C = 5.15 \times 31.8 = 163.8 V$

(e) The power factor $pf = \cos 51.9^\circ = 0.617$ lagging.

(f) The apparent power $= P_{app} = 100 \times 5.15 = 515 VA$.

(g) The average power $= P_{avg} = VI \cos 51.9^\circ = 317.77 W$.

4. A current 0.9 A flows through a series combination of a resistor of 120 Ω and a capacitor of reactance 250 Ω. Find the impedance, power factor, supply voltage, voltage across resistor, voltage across capacitor, apparent power, active power and reactive power.

Taking current as the reference phasor $I = 0.9 \angle 0^\circ A$

Impedance $Z = 120 - j250 = 277.3 \angle -64.4^\circ \Omega$

power factor $= pf = \cos \theta = \cos(-64.4^\circ) = 0.432$ leading.

Supply voltage $= V = IZ = (0.9 \angle 0^\circ)(277.3 \angle -64.4^\circ) = 249.6 \angle -64.4 V$

Voltage across resistor $= V_R = IR = (0.9 \angle 0^\circ) \times 120 = 108 \angle 0^\circ V$

Voltage across capacitor $= V_C = IX_C = (0.9 \angle 0^\circ) \times (250 \angle -90^\circ) = 225 \angle -90^\circ V$

Apparent power $= P_{app} = VI = 249.6 \times 0.9 = 224.6 VA$.

Actual power $= P_a = VI \cos \theta = 249.6 \times 0.9 \times \cos 64.4^\circ = 97.06 W$

Reactive power $= P_r = VI \sin \theta = 249.6 \times 0.9 \times \sin 64.4^\circ = 202.59 VAR$

5. An ac sinusoidal voltage $V = (160 + j120) V$ is applied to a circuit. The resulting current is $I = (-4 + j10) A$. Find the impedance of the circuit and state whether it is inductive or capacitive. Also find the power factor, active power and reactive power.

$V = 160 + j120$

$= 200 \angle 36.87^\circ V$.

$I = -4 + j10$

$= 10.77 \angle 111.8^\circ$ A.

6. When a two element series circuit is connected across an ac source of frequency 50Hz, it offers an impedence $Z = (10 + j10)\Omega$. Determine the value of the two element

$Z = (R + jX_L) = (10 + j10)\Omega$.

$R = 10\Omega \qquad X_L = 10\Omega$

$$L = \frac{X_L}{2\pi f} = \frac{10}{2\pi \times 50} = 0.318H = 31.8mH$$

7. When a two-element parallel circuit is connected across an ac source of frequency 50Hz, it offers an impedence $Z = (10 - j10)\Omega$. Determine the values of two elements.

$$Y = G + jB = \frac{1}{Z} = \frac{1}{10 - j10} = \frac{1}{14.14 \angle -45^\circ} = 0.0707 \angle 45^\circ$$

$S = (0.05 + j0.05)S$.

$$G = 0.05S - R = \frac{1}{G} = \frac{1}{0.05} = 20\Omega$$

$B = 0.05S$

$$C = \frac{B}{\omega} = \frac{0.05}{2\pi f} = 159\mu F.$$

Page No:
Date: / /

Three phase circuit

1. Advantages of 3 phase power.

1. Three-phase transmission lines require less conductor material. Since the phasor sum of currents in all the phases is zero, there is substantial saving by eliminating the return conductor or replacing it by a single neutral conductor of comparatively small size.
2. For a given frame size, a three-phase machine gives a higher output than a single-phase machine.
3. The power in a single-phase system pulsates at twice the line frequency. However, the sum of powers in the three phases in a three-phase system remains constant. Therefore, a three-phase motor develops a uniform torque, whereas a single phase motor develops pulsating torque.
4. Since the three phase supply can generate a rotating field the three phase induction motors are self-starting.
5. The three-phase system can be used to supply domestic as well as industrial power.
6. The voltage regulation in three phase system is better than that in single-phase supply.

2. Generation of 3ϕ EMF :-

1. In the 3ϕ System, there are three equal voltages of the same frequency but displaced from one another by 120° electrical.
2. These voltages are produced by a three-phase generator which has three identical winding or phases displaced 120° electrical apart.

3. When these winding are rotated in a magnectic field, emf is induced in each winding or phase. These emf are of the same magnitude and frequency but are displaced from one another by 120° electrical.

4. Consider three electrical coil $a_1 a_2$ $b_1 b_2$ and $c_1 c_2$ mounted on the same axis but displaced from each other by 120° electrical.

5. Let the three coils be rotated in an anticlockwise direction in a bipolar magnetic field with an angular velocity of ω radians/sec.

6. Here $a_1 b_1 c_1$ are the start terminal and a_2, b_2 and c_2 are the end terminal of the coil.

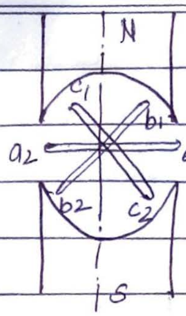
7. When the coil $a_1 a_2$ is in the position AB, the magnitude and the direction of the emf's induced in the various coil is as under.

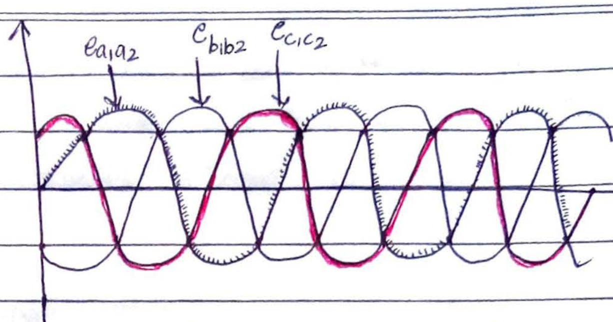
a) EMF induced in coil a_1, a_2 is zero and is increasing in the positive direction. This is indicated by $e_{a_1 a_2}$ wave.

b) The coil $b_1 b_2$ is 120° electrically behind coil $a_1 a_2$. The emf induced in this coil is negative and is approaching maximum negative value. This is shown by the $e_{b_1 b_2}$ wave.

c) The coil $c_1 c_2$ is 240° electrically behind $a_1 a_2$ or 120° electrically behind coil $b_1 b_2$. The emf induced in this coil is positive and is decreasing. This is indicated by wave $e_{c_1 c_2}$.

Thus it is apparent that the emf's induced in the three coil are of the same magnitude and frequency but displaced by 120° electrical from each other.





Accordingly, the instantaneous value of the emfs induced in the coil a_1a_2, b_1b_2 and c_1c_2 may be given as

$$e_{a_1a_2} = E_{max} \sin \omega t$$
$$e_{b_1b_2} = E_{max} (\sin \omega t - 120^\circ)$$
$$e_{c_1c_2} = E_{max} (\sin \omega t - 240^\circ).$$

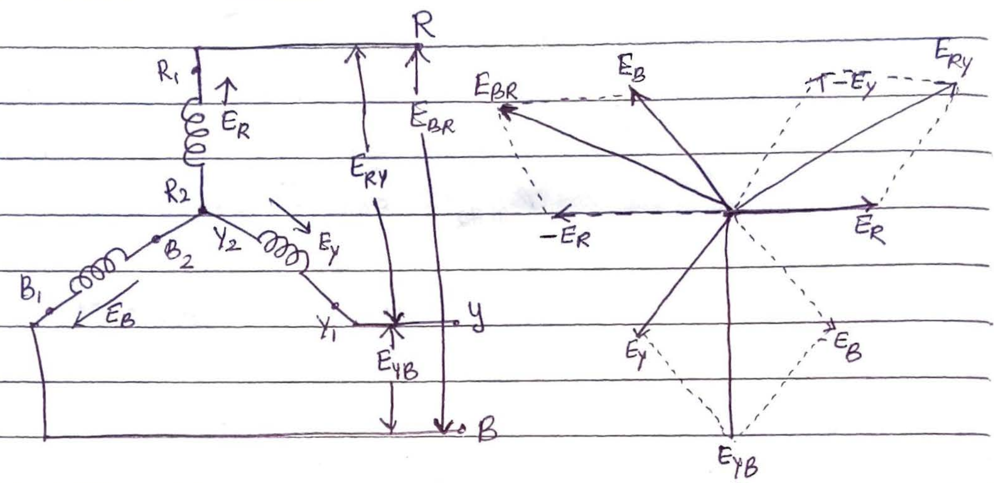
if $t=0$ corresponds to the instant when the voltage or emf of coil a_1a_2 passes through zero and increases in positive direction.

$$E_{a_1a_2} = E_{max} = E_p$$
$$e_{b_1b_2} = E_{max} \sin(-120^\circ) = E_p \angle -120^\circ$$
$$e_{c_1c_2} = E_{max} \sin(-240^\circ) = E_p \angle 120^\circ$$

8. At any Instant, prove the algebric sum of the three emf is zero.

$$\text{Resultant} = e_{a_1a_2} + e_{b_1b_2} + e_{c_1c_2}$$
$$= E_m \sin \omega t + E_m \sin(\omega t - 120) + E_m \sin(\omega t + 120)$$
$$= E_m[\sin \omega t + \sin(\omega t - 120) + \sin(\omega t + 120)]$$
$$= E_m[\sin \omega t + \sin \omega t \cos 120 - \sin 120 \cos \omega t + \sin \omega t \cos 120 + \cos \omega t \sin 120]$$
$$= E_m[\sin \omega t + 2 \sin \omega t \cos 120]$$
$$= E_m\left[\sin \omega t + 2 \sin \omega t \left(-\frac{1}{2}\right)\right] = 0$$

4. In the star connection find the relation between line and phase values of current and voltage and also derive equation for 3ϕ power.



* This system is obtained by joining together similar ends, either the start or the finish. The other ends are joined to the line wire.
* The common point N at which similar ends are connected is called neutral or start point.
* The three phases be either numbered as 1, 2, 3 or a, b, c or R, Y, B. R, Y, B indicate the three natural colors Red, Yellow, Blue.
* There are only two possible phase sequences, namely RYB and RBY. By convention sequence RYB is taken as positive and RBY as negative.
* potential difference between two outer R and Y or E_{RY} is the phasor difference of phase emfs E_R and E_Y or phasor sum of phase emfs E_R and $(-E_Y)$

(ie) $E_{RY} = E_R - E_Y$

$E_{RY} = E_R + (-E_Y)$

Since phase angle between phasor E_R and $(-E_Y)$ is 60°. From the phasor diagram,

$$E_{RY} = \sqrt{E_R^2 + E_Y^2 + 2E_R E_Y \cos 60^\circ}$$

Page No:
Date: / /

[Note:- Because of trignometric vector addition :- Two vectors of length a and b make an angle θ with each other when placed tail to tail, the magnitude of the resultan is given by

$$r = \sqrt{a^2+b^2+2ab\cos\theta}]$$

* The voltage between the line and neutral point (ie) voltage across the phase winding is called phase voltage.

Let $E_R = E_Y = E_B = E_p$ (phase voltage)

Line voltage $E_{RY} = \sqrt{E_p^2 + E_p^2 + 2E_p \cdot E_p \cos 60^\circ}$

$$= \sqrt{E_p^2+E_p^2+2E_p^2\left(\tfrac{1}{2}\right)}$$

$$= \sqrt{3E_p^2}$$

$$E_{RY} = \sqrt{3}\,E_p.$$

Similarly potential difference between outer Y and B.

$$E_{YB} = \sqrt{3}\,E_p$$

$$E_{BR} = \sqrt{3}\,E_p.$$

In star connected system each line conductor is connected to seperate phase, so current flowing through the lines and phase are same.

Line current I_L = phase current I_p.

* If the phase current has a phase difference of ϕ with the phase voltage

power output per phase = $E_p I_p \cos\theta$.

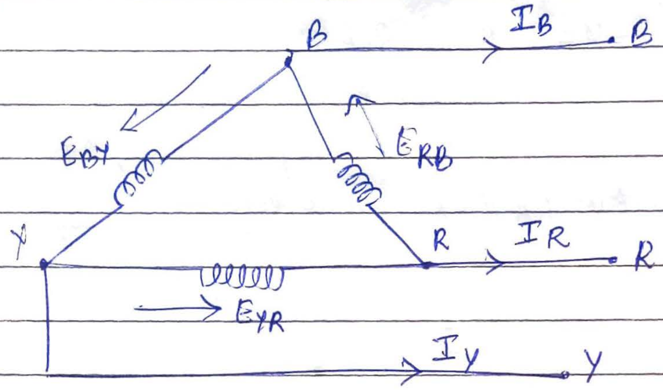
total power output, $P = 3E_p I_p \cos\phi = 3\dfrac{E_L}{\sqrt{3}} I_L \cos\theta$

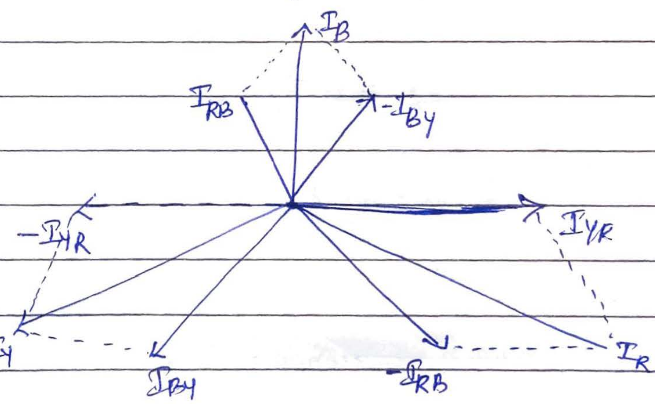
$$P = \sqrt{3}\,E_L I_L \cos\phi$$

power = √3 x line voltage x line current x power factor.

apparent power of 3φ

5. In the delta connection find the relation between line and phase values of current and voltage and also derive equation for 3ϕ Power.





* When the starting end of one coil is connected to the finishing end of another delta is obtained.
* Line current

$$I_R = I_{YR} - I_{RB} \quad \text{(Vector difference)}$$
$$= I_{YR} + (-I_{RB}) \quad \text{(Vector sum)}$$

As the phase angle between current I_{YR} and $-I_{RB}$ is $60°$.

$$\therefore\ I_R = \sqrt{I_{YR}^2 + I_{RB}^2 + 2 I_{RB} I_{RB} \cos 60°}$$

For a balanced load, the phase current in each winding is equal and let it be $= I_p$

Page No:

Date: / /

Line current $I_R = \sqrt{I_P^2 + I_P^2 + 2 I_P I_P \times 0.5}$

$= \sqrt{3}\, I_P.$

Similarly line current $I_Y = I_{BY} - I_{YR} = \sqrt{3}\, I_P.$

line current $I_B = I_{RB} - I_{BY} = \sqrt{3}\, I_P$

In a delta network there is only one phase between any pair of line outers, so the potential difference between the line outers, called the line voltage, is equal to phase voltage

line voltage E_L = phase voltage E_P.

power output per phase = $E_P I_P \cos\phi$

where $\cos\phi$ is the pf. of the load.

Total power output $P = 3 E_P I_P \cos\phi$

$= 3 E_L \frac{I_L}{\sqrt{3}} \cos\phi$

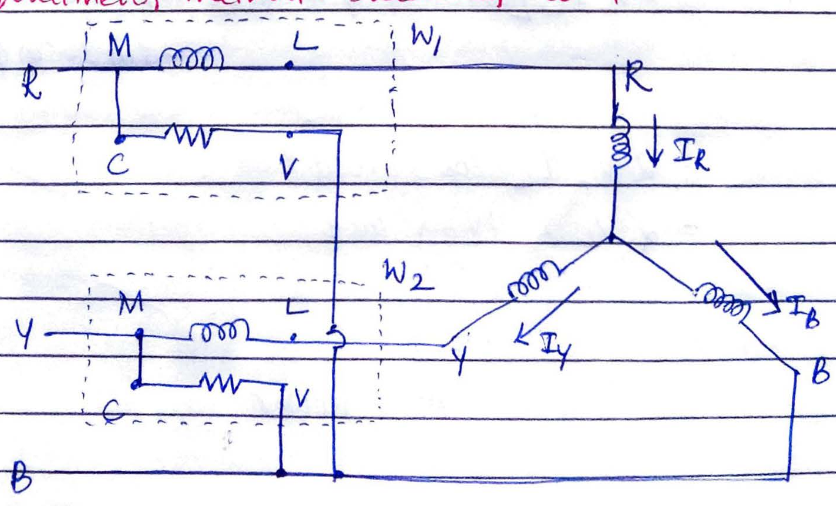
$= \sqrt{3}\, E_L I_L \cos\phi$

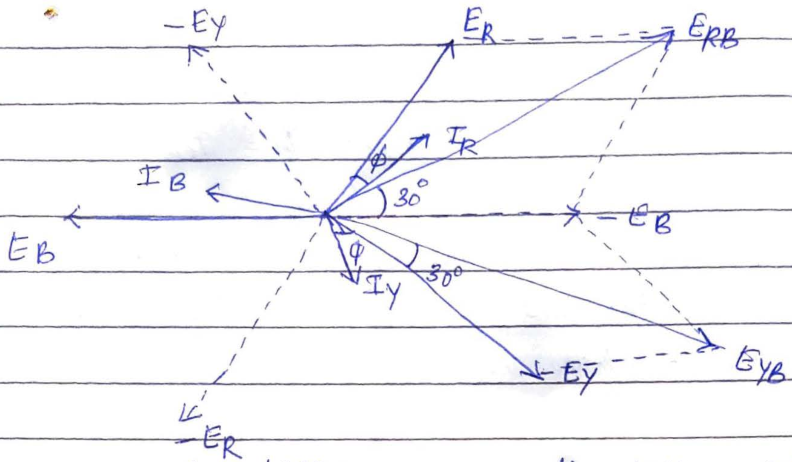
Total power output = $\sqrt{3}$ x line voltage x line current x Pf

apparent power of 3 phase delta connected system $= 3 E_P I_P$

$= \sqrt{3}\, E_L I_L$

6 Measurement of Power in 3 phase circuits: Two wattmeter method - Balanced load.





The potential difference across the voltage coil of W_1 is

$$E_{RB} = E_R - E_B$$

This E_{RB} is the vectorial resultant of E_R and $-E_B$.

Reading in $W_1 = I_R E_{RB} \cos(30° - \phi)$

$$W_1 = V_L I_L \cos(30° - \phi)$$

Similarly,

$$E_{YB} = E_Y - E_B.$$

$$\therefore W_2 = I_Y E_{YB} \cos(30 + \phi).$$

$$W_2 = V_L I_L \cos(30 + \phi)$$

$$W_1 + W_2 = V_L I_L [\cos(30 - \phi) + \cos(30 + \phi)]$$

$$= V_L I_L [\cos 30 \cos\phi + \sin 30 \sin\phi + \cos 30 \cos\phi - \sin 30 \sin\phi]$$

$$= 2 V_L I_L \cos 30 \cos\phi$$

$$= \cancel{2} V_L I_L \left(\frac{\sqrt{3}}{\cancel{2}}\right) \cos\phi$$

$$= \sqrt{3} V_L I_L \cos\phi$$

= Total 3 phase power

Page No:
Date: / /

7. How power factor is obtained from two wattmeter reading.

For a balanced, lagging Pf load.

$$W_1 = V_L I_L \cos(30-\phi)$$
$$W_2 = V_L I_L \cos(30+\phi)$$

$$W_1 + W_2 = \sqrt{3}\, V_L I_L \cos\phi$$
$$W_1 - W_2 = V_L I_L \left[\cos(30-\phi) - \cos(30+\phi)\right]$$

$$= V_L I_L \left[\cos 30 \cos\phi + \sin 30 \sin\phi - \cos 30 \cos\phi + \sin 30 \sin\phi\right]$$

$$= V_L I_L \left[2 \sin 30 \sin\phi\right]$$
$$= V_L I_L \left[2 \times \frac{1}{2} \times \sin\phi\right]$$

$$\therefore W_1 - W_2 = V_L I_L \sin\phi$$

$$\frac{W_1 - W_2}{W_1 + W_2} = \frac{V_L I_L \sin\phi}{\sqrt{3}\, V_L I_L \cos\phi} = \frac{\tan\phi}{\sqrt{3}}$$

$$\tan\phi = \frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2}$$

$$\phi = \tan^{-1}\left[\frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}\right]$$

$$\text{Pf } \cos\phi = \cos\left[\tan^{-1}\left[\frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2}\right]\right]$$

Problems :-

1. A 3ϕ 400V, motor takes an input of 40 KW at 0.45 pf lag. Find the reading of each of the two single phase wattmeters connected to measure the input.

$V_L = 400V$

$P_{in} = 40 KW$

$\cos\phi = 0.45$ lag

$P_{in} = \sqrt{3} \times V_L \times I_L \times \cos\phi$

$40 \times 10^3 = \sqrt{3} \times 400 \times I_L \times 0.45$

$I_L = 128.3 A$

$V_L I_L = 51320$

$\phi = \cos^{-1} 0.45 = 63.256$

$W_1 = V_L I_L \cos(30 - \phi) = 51320 \cos(30^\circ - 63.256)$

$= 42915.2 W$

$W_2 = V_L I_L \cos(30 + \phi) = 51320 \cos(30 + 63.25)$

$= -2915.2 W.$

2. A 3ϕ delta connected balanced load consumes a power of 60 KW taking a current of 200A. lagging at line voltage of 400V, 50Hz. Find the parameter in each phase.

$P_{in} = 60 KW$

$V_L = 400V$

$I_L = 200A$

$P_{in} = \sqrt{3} V_L I_L \cos\phi$

Page No:

Date: / /

$$60 \times 10^3 = \sqrt{3} \times 200 \times 400 \times \cos\phi$$

$$\therefore \cos\phi = \frac{600 \times 10^3}{\sqrt{3} \times 200 \times 400} = 0.433$$

$$\phi = 64.34^\circ$$

For delta,

$$V_{ph} = V_L = 400V$$

$$I_L = \sqrt{3} I_{ph}$$

$$I_{ph} = \frac{I_L}{\sqrt{3}} = \frac{200}{\sqrt{3}} = 115.47A$$

$$Z_{ph} = \frac{V_{ph}}{I_{ph}} = \frac{400}{115.47} = 3.464\Omega$$

$$Z_{ph} = 3.464 \angle 64.34$$

$$= 1.5 + j\,3.122\ \Omega$$

$$R_{ph} = 1.5\Omega$$

$$X_{L\,ph} = 3.122\Omega$$

$$X_L = 2\pi f L$$

$$L = \frac{\cancel{(2\pi f}}{X_L} = L = \frac{X_L}{2\pi f} = \frac{3.122}{2 \times \pi \times 50}$$

$$L = 9.93\ mH/phase$$

3. A balanced star-connected load of $(8+j6)$ Ω per phase is connected to a 3 phase 230V supply. Find the line current, power factor, power, reactive volt-ampere and total volt ampere

$$Z_p = (8+j6)\Omega$$

$$E_L = 230V$$

(1) For star connected system

Phase voltage $E_p = \frac{E_L}{\sqrt{3}}$

$= \frac{230}{\sqrt{3}} = 132.79V$

Magnitude of $Z_p = \sqrt{(8^2+6^2)} = 10\Omega$

Phase current $= I_p = \frac{E_p}{Z_p}$

$= \frac{132.79}{10} = 13.27A$

For delta connection,

$I_p = \frac{I_L}{\sqrt{3}}$

Current in each phase $= I_p = \frac{72.4}{\sqrt{3}} = 41.8A$