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Module - 2

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Taylor's Series

$$y(x) = f(a) + (x-a) f_1(a) + \frac{(x-a)^2}{2!} y_1(a) + \frac{(x-a)^3}{3!} y_3(a) + \cdots$$

Maclaurin's Series :-

$$y(x) = y(0) + x y_1(0) + \frac{x^2}{2!} y_2(0) + \frac{x^3}{3!} y_3(0) + \cdots$$

$x = a$

① Obtain the taylor's series of $\log x$ about $x = 1$ upto the terms containing 4th degree and hence obtain $\log(1.1)$

$$y(x) = f(a) + (x-a) f_1(a) + \frac{(x-a)^2}{2!} y_1(a) + \frac{(x-a)^3}{3!} y_3(a) + \frac{(x-a)^4}{4!} y_4(a)$$

$$y = \log x$$

$$y(x) = y(1) + (x-1) y_1 + \frac{(x-1)^2}{2!} y_2(1) + \frac{(x-1)^3}{3!} y_3(1) + \frac{(x-1)^4}{4!} y_4(1)$$

$y(1) = \log(1) \Rightarrow \boxed{y(1) = 0}$

$y_1 = \frac{1}{x} \Rightarrow \boxed{y_1(1) = 1}$

$y_2 = -\frac{1}{x^2} \Rightarrow \boxed{y_2(1) = -1}$

0.05
0.003
0.00025

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$$y_3 = \frac{2}{x^3} \Rightarrow \boxed{y_3(1) = 2}$$

$$y_4 = \frac{-6}{x^4} \Rightarrow \boxed{y_4(1) = -6}$$

$$y(x) = 0 + (x-1)\,1 + \frac{(x-1)^2}{2}(-1) + \frac{(x-1)^3}{\cancel{6}\,3}(\cancel{2}) + \frac{(x-1)^4}{\cancel{24}\,4}(-\cancel{6})$$

$$y(x) = (x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \frac{(x-1)^4}{4}$$

$$y(1.1) = 1.1 - 1$$

$$y(1.1) = (0.1) - \frac{(0.1)^2}{2} + \frac{(0.1)^3}{3} - \frac{(0.1)^4}{4}$$

$$y(1.1) = 0.0953$$

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(2) Expand $\tan^{-1}x$ in powers of $x-1$ upto 4^{th} degree

formula:- $y(x) = f(a) + (x-a) f_1(a) + \frac{(x-a)^2}{2!} y_2(a) + \frac{(x-a)^3}{3!} y_3(a) + \frac{(x-a)^4}{4!} y_4(a)$

Sol:- $y = \tan^{-1}(x)$, $x = a = 1$

$$y(1) = \tan^{-1}1 = \pi/4$$

$$y_1 = \frac{1}{1+x^2} \Rightarrow y_1(1) = \frac{1}{1+1}$$

$$\boxed{y_1(1) = \frac{1}{2}}$$

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$$(1+x^2)y_1 = 1$$

$$(1+x^2)y_2 + 2xy_1 = 0$$

$$(1+1)y_2(1) + 2(1)(1/2) = 0$$

$$2y_2(1) = 0$$

$$\cancel{\boxed{y_2(1) = 0}} \quad \boxed{y_2(1) = -1/2}$$

$$(1+x^2)y_3 + 2xy_2 + 2xy_2 + 2y_1 = 0$$
$$(1+x^2)y_3 + 4xy_2 + 2y_1 = 0$$
$$(1+1)y_3(1) + \cancel{4}^{2}(1)(-1/\cancel{2}) + \cancel{2}(1/\cancel{2}) = 0$$

$$\boxed{y_3(1) = 1/2}$$

$$(1+x^2)y_4 + 2xy_3 + 4xy_3 + 4y_2 + 2y_2 = 0$$

$$(1+x^2)y_4 + 6xy_3 + 6y_2 = 0$$
$$(1+1)y_4(1) + 6(1)(1/2) + 6(-1/2) = 0$$

$$\boxed{y_4(1) = 0}$$

$$y(x) = y(1) + (x-1)y_1(1) + \frac{(x-1)^2}{2!}y_2(1)$$
$$+ \frac{(x-1)^3}{3!}y_3(1) + \frac{(x-1)^4}{4!}y_4(1)$$

② Obtain the Maclaurin's expression of $\sin^{-1}(x)$ upto the term containing x^5

$y = \sin^{-1}(x)$

$y(x) = y(0) + xy_1(0) + \frac{x^2}{2!} y_2(0) + \frac{x^3}{3!} y_3(0)$

$+ \frac{x^4}{4!} y_4(0) + \frac{x^5}{5!} y_5(0)$

$y = \sin^{-1} x$ $\boxed{y(0) = 0}$

$y_1 = \frac{1}{\sqrt{1-x^2}}$ $\Rightarrow$ $\boxed{y_1(0) = 1}$

$\frac{1}{\sqrt{1-0}}$ $\frac{1}{1} = 1$

$(1-x^2)\, y_1^2 = 1$

$(1-x^2)\, 2y_1y_2 - 2xy_1^2 = 0$

$\boxed{y_2(0) = 0}$

$y_1 (1-x^2)\, \cancel{2}y_2 - \cancel{2}xy_1 = 0$

$(1-x^2)\, \cancel{2}y_3 + \cancel{2}y_2(-2x) - \cancel{2}xy_2 - \cancel{2}y_1 = 0$

$(1-x^2)y_3 - 3xy_2 - y_1 = 0$

$(1-x^2)\, y_3(0) - 3x\, y_2(0) - y_1(0) = 0$

$y_3 = \frac{3x^2 y_2 + y_2}{(1-x^2)}$

$y_3 = 3$

$\boxed{y_3(0) = 1}$

~~$y_4(0) =$~~

$(1-x^2)\, y_4 - 2xy_3 - 3xy_3 - 3y_2 - y_2 = 0$

$(1-x^2)y_4 - 3xy_3 - 4y_2 = 0$

$(1-x^2)\, y_4(0) - 3x\, y_3(0) - 4y_2(0) = 0$

$y_4 = \frac{5xy_3 + 4y_2}{(1-x^2)} = \frac{0+0}{1-0} = 0$

$\boxed{y_4(0) = 0}$

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$$(1-x^2)y_5 - 5xy_4 - 5y_3 - 4y_3 = 0$$

$$(1-x^2)y_5 - 5yy_3 - 5xy_4 - 4y_3 = 0$$

$$(1-x^2)y_4 - 5xy_4 - 9y_3 = 0$$

$$y_5 = \frac{7xy_4 + 9y_3}{(1-0)}$$

$$y_5 = 9/1$$

$$\boxed{y_5 = 9}$$

$$y(x) = 0 + x + 0 + \frac{x^3}{3!} + 0 + \frac{x^5}{5!} \cdot 9$$

$$= x + \frac{x^3}{6} + \frac{3x^5}{40}$$

(3) Expand $\log(1+\sin x)$ upto x^4

$$y = \log(1+\sin x)$$

$$y_1 = \frac{1}{1+\sin x}$$

$$y(0) = 0$$

$$y_1 = \frac{1}{1+\sin x}(\cos x)$$

$$y_1 = \frac{\cos x}{1+\sin x}$$

$$y_1(0) = \frac{1}{1+0} \Rightarrow y_1(0) = 1$$

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~~(1+sinx)~~

$\cancel{y_2 = -\sin x\,(1+\sin x)\,\cos x\,(\cos x)}$

$\cancel{= \sin x}$

$(1+\sin x)y_2 + y_1(\cos x) = -\sin x$

$1+\sin x\, y_2 + \cos x = -\sin x$

$y_2 = \dfrac{-\tan x}{1+\sin x}$ $\qquad y_2 + 1 = 0$

$\boxed{y_2 = -1}$

$(1+\sin x)y_3 + 2y_2\cos x - y_1\sin x = -\cos x$

$x = 0,\quad y_3(0) - 2 - 0 = -1$

$\therefore\ y_3(0) = 1$

$(1+\sin x)y_4 + \cos x\, y_3 + 2(-y_2\sin x + \cos x\, y_3)$
$- (y_1\cos x + \sin x\, y_2) = \sin x$

$x = 0,\quad y_4(0) + 3 - 0 - 1 = 0$

$\therefore\ y_4(0) = -2$

$y(x) = y(0) + x y_1(0) + \dfrac{x^2}{2!}y_2(0) + \dfrac{x^3}{3!}y_3(0)$
$+ \dfrac{x^4}{4!}y_4(0)$

$4\times3\times2\times1$ $\qquad = 0 + x + \dfrac{x^2}{2}(-1) + \dfrac{x^3}{3!} + \dfrac{x^4}{4!}(-2)$

$\cancel{\dfrac{x^4(-2)}{24}}$

$3\ \ 12$

$= 0 + x - \dfrac{x^2}{2} + \dfrac{x^3}{6} - \dfrac{x^4}{12}$

$$\boxed{\log(1+\sin x) = x - \frac{x^2}{2} + \frac{x^3}{6} - \frac{x^4}{12}}$$

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Expand log (secx) upto the terms containing x^6 using Maclaurin's series :-

(1) log (secx) upto x^6

$y = \log(\sec x)$

$y_1 = \frac{1}{\sec x} \sec x . \tan x$

$y_1 = \frac{\cancel{\sec x} . \tan x}{\cancel{\sec x}}$

$y_1 (0) = 0$

$y_2 = \sec^2 x \Rightarrow (1 + \tan^2 x) = ((1+y)^2) = 1+0$

$y_2 = 1$

$y_3 = 2 \sec x . \sec x . \tan x$

$y_3 = 2 \sec^2 x \tan x$

$y_3 = 0$

$y_3 = 2 y_1 y_2$

$y_3 = 2(0)(1)$

$y_3 = 0$

$y_4 = 2y_2^2 + y_1 y_3$

$y_4 = 2$

$\cancel{y_5 = 4y_2 + y_3 + y_3 . y_2 + y_1 y_4}$

$\cancel{y_5 = 0 + 0 + 0}$

$y_5 = 0$

$6\times5\times4\times3\times2$ $\quad$ $\overset{120}{360\times2} = 720$

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$$\cancel{y_5 = 2y_2y_3 + y_3y_2 + y_1y_4 + 2y_3y_3 + y_2y_4}$$

$$\cancel{y_5 = 2y_2y_3 + y_3y_2 + y_1y_4 + 2y_3^2}$$

$$y_5 = 2(y_1y_4 + y_2y_3 + 2y_2y_3)$$
$$= 2y_1y_4 + 6y_2y_3$$
$$y_5(0) = 0$$

$$y_6 = 2(y_1y_5 + y_2y_4) + 6(y_2y_4 + y_3^2)$$
$$y_6 = 2y_1y_5 + 8y_2y_4 + 6y_3^2$$
$$\therefore\ y_6(0) = 16$$

$$y(x) = y(0) + xy_1(0) + \frac{x^2}{2!}y_2(0) + \frac{x^3}{3!}y_3(0) + \frac{x^4}{4!}y_4(0)$$
$$+ \frac{x^5}{5!}x_5(0) + \frac{x^6}{6!}x_6(0)$$

$$= 0 + 0 + \frac{x^2}{2} + \frac{x^3}{6}(0) + \frac{x^4}{\cancel{24}_{12}}\cdot\cancel{2} + \frac{x^5}{5!}(0) + \frac{x^6}{6!}(16)$$

$$= 0 + 0 + \frac{x^2}{2} + 0 + \frac{x^4}{12} + 0 + \frac{x^6}{45}$$

$$= \frac{x^2}{2} + \frac{x^4}{12} + \frac{x^6}{45}$$

\# $\tan\left(\frac{\pi}{4} + x\right)$

$$y = \tan\frac{\pi}{4}$$

$$y(0) = 1$$

$y_2 = \sec^2 \frac{\pi}{4} \Rightarrow 1 + \tan^2 \frac{\pi}{4} \Rightarrow 1$

$y_1(0) = 2$

$y_2 = \cancel{1} + 2y_1 y_2$

$= (1 + 2(2)(1))$

$y_2 = 4$

$y_3 = 2yy_2 + 2y_1 y_1$

$= 2y_2 y + 2y_1^2 + 2$

$2(4) + 2(4)$

$y_3 = 16$

$y_4 = 2yy_3 + 2y_2^2 + 4y_1 y_2$

$y_4 = 2(1)(16) + 2(16) + 4(1)(4)$

$= 32 + 32 + 16$

$y_4 = 80$

$= 1 + 2x + \frac{x^2}{\cancel{2}} \cdot \cancel{4}^{2} + \frac{x^3}{\cancel{6}_{3}} \cdot \cancel{16}^{8} + \frac{x^4}{\cancel{24}\ \cancel{12}\ \cancel{6}\ 3} \cdot \cancel{80}\ \cancel{40}\ \cancel{20}\ 10$

$= 1 + 2x + 2x^2 + 8 \cdot \frac{x^3}{3} + \frac{10}{3} x^4$

L-Hospital's Rule

$$\boxed{\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}}$$

$$\lim_{x \to 0} \frac{x^3}{x^2} \left(\frac{0}{0}\right)$$

$$= \frac{3x^2}{2x} \left(\frac{0}{0}\right)$$

$$= \frac{6x}{2} \left(\frac{0}{2}\right)$$

$$\Rightarrow \frac{6x}{2} = 0$$

Indeterminate form

Evaluate the following limits

1) $$\lim_{x \to 0} \frac{xe^x - \log(1+x)}{x^2} \left(\frac{0}{0}\right)$$

$$\lim_{x \to 0} \frac{xe^x + e^x - 1/1+x}{2x} \left(\frac{0}{0}\right)$$

$$= \lim_{x \to 0} \frac{xe^x + e^x + e^x + 1/(1+x)^2}{2} = 0+1+1+1 = \frac{3}{2}$$

2) $$\lim_{x \to 0} \frac{1-\cos x}{x \log(1+x)} \left(\frac{0}{0}\right)$$

$$\lim_{x \to 0} \frac{0 + \sin x}{x \cdot \frac{1}{(1+x)} + \log(1+x)} \left(\frac{0}{0}\right)$$

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$$= \lim_{x\to 0} \frac{\cos x}{x\left(\frac{-1}{(1+x)^2}\right)+\left(\frac{1}{1+x}\right)+\frac{1}{1+x}} = \frac{1}{2}$$

$$= \frac{1}{2}$$

3) $$K = \lim_{x\to \pi/2} \frac{\log(\sin x)}{\left(\frac{\pi}{2}-x\right)^2} \quad \left(\frac{0}{0}\right)$$

$$= \lim_{x\to\pi/2} \frac{\frac{1}{\sin x}\cdot \cos x}{-2\left(\frac{\pi}{2}-x\right)} = \lim_{x\to\pi/2} \frac{\cot x}{-2\left(\frac{\pi}{2}-x\right)} \quad \left(\frac{0}{0}\right)$$

$$= \lim_{x\to\pi/2} \frac{-\operatorname{cosec}^2 x}{-2(-1)} = \frac{-1}{2}$$

4) $$\lim_{x\to\pi/2} \frac{\log(x-\pi/2)}{\tan x} \quad \left(\frac{\infty}{\infty}\right)$$

$$= \lim_{x\to\pi/2} \frac{\left(\frac{1}{x-\pi/2}\right)}{\sec^2 x} = \lim_{x\to\pi/2} \frac{\cos^2 x}{(x-\pi/2)}$$

$$= -\frac{2\cos x\cdot \sin x}{1} = -\frac{2(1)(0)}{1}$$

$$= 0$$

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5) $\lim_{x\to 1}\left(\frac{x}{x-1}-\frac{1}{\log x}\right)$ $(\infty-\infty)$

$$k=\lim_{x\to 1}\frac{x\log x-(x-1)}{(x-1)\log x}\ \left(\frac{0}{0}\right)$$

$$=\lim_{x\to 1}\frac{\left(x\cdot 1/x+\log x\right)-1}{(x-1)\,1/x+\log x}$$

$$\lim_{x\to 1}\frac{\frac{\cancel{x}+x\log x-\cancel{x}}{\cancel{x}}}{\frac{(x-1)+x\log x}{\cancel{x}}}$$

$$=\lim_{x\to 1}\frac{x\log x}{(x-1)+x\log x}\ \left(\frac{0}{0}\right)$$

$$=\lim_{x\to 1^-}\frac{x\cdot\frac{1}{x}+\log x}{1+x\frac{1}{x}+\log x}=\lim_{x\to 1}\frac{x+x\log x}{2x+x\log x}=\frac{1}{2}$$

6) $\lim_{x\to 2}\left(\frac{1}{x-2}-\frac{1}{\log(x-1)}\right)$ $(\infty-\infty)$

$$k=\lim_{x\to 2}\frac{\log(x-1)-(x-2)}{(x-2)\log(x-1)}\ \left(\frac{0}{0}\right)$$

$$\lim_{x\to 2}\frac{\frac{1}{(x-1)}-1}{(x-2)\frac{1}{(x-1)}+\log(x-1)}\ \left(\frac{0}{0}\right)$$

$$=\lim_{x\to 2}\frac{1-(x-1)}{(x-2)+(x-1)\log(x-1)}$$

$\left(\infty - \frac{0}{0}\right)$

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$$\lim_{x\to 2} \frac{0-1}{1+(x-1)\frac{1}{x-1}+\log(x-1)} = \frac{-1}{2}$$

7) $\lim_{x\to 0}\left(\frac{1}{x} - \frac{\log(1+x)}{x^2}\right) \quad \left(\infty - \frac{0}{0}\right)$

$$\lim_{x\to 0} \frac{\left(\frac{1}{x}\right)^2 \quad \frac{-1}{x^2} - \frac{\frac{1}{1+x}}{2x}}{}$$

$$\lim_{x\to 0}\left(-\frac{1}{x^2} - \frac{1/1+x}{2x}\right)$$

$$-\left(\frac{1}{x^2} + \frac{1/1+x}{2x}\right) \quad (\infty+\infty)$$

$\frac{1}{1+x}$

$\lim_{x\to 0}$

7) $\lim_{x\to 0}\left(\frac{1}{x} - \frac{\log(1+x)}{x^2}\right) \quad \left(\infty - \frac{0}{0}\right)$

$$\lim_{x\to 0}\left(\frac{x - \log(1+x)}{x^2}\right) \quad \left(\frac{0}{0}\right)$$

$$\lim_{x\to 0}\left(\frac{1-\frac{1}{1+x}}{2x}\right)\left(\frac{0}{0}\right)$$

$$\lim_{x\to 0}\left(\frac{\frac{1}{(1+x)^2}}{2}\right) = \frac{1}{2}$$

8) $\lim_{x\to \pi/2}\left(2x\tan x - \pi\sec x\right) \quad (\infty - \infty)$

$$\lim_{x\to \pi/2}\left(2x\sec^2 x + \tan x \cdot 2 - \pi\cdot\sec x\cdot\tan x\right) = ($$

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$$\lim_{x\to\pi/2}\left(2x\tan x - \pi\sec x\right)$$

$$\lim_{x\to\pi/2}\left(\frac{2x\sin x}{\cos x} - \pi\cdot\frac{1}{\cos x}\right) = (\infty - \infty)$$

$$\lim_{x\to\pi/2}\left(\frac{2x\sin x - \pi}{\cos x}\right)\left(\frac{0}{0}\right)$$

$$= \lim_{x\to\pi/2}\frac{2x\cos x + 2\sin x - 0}{-\sin x}$$

$$= -2$$

(9) $$\lim_{x\to 0}\left(\frac{\tan x - x}{x^2\cdot\tan x}\right)\cdot\cancel{\tan x}\quad\left(\frac{0}{0}\right)$$

$$\lim_{x\to 0}\frac{\tan x - x}{x^2\cdot\left(\frac{\tan x}{x}\right)\cdot x}$$

$$= \lim_{x\to 0}\frac{\tan x - x}{x^3}\cdot\lim_{x\to 0}\frac{x}{\tan x}$$

$$= \lim_{x\to 0}\frac{\tan x - x}{x^3}\quad\left(\frac{0}{0}\right)$$

$$= \lim_{x\to 0}\frac{\sec^2 x - 1}{3x^2}$$

$$= \lim_{x\to 0}\frac{\tan^2 x}{3x^2}$$

$$\frac{1}{3}\lim_{x\to 0}\left(\frac{\tan x}{x}\right)^2$$

$$\frac{1}{3}\cdot 1 = \frac{1}{3}$$

(10) $\lim\limits_{x\to 0} \dfrac{x^2+2\cos x-2}{x\sin^3 x} \quad \left(\dfrac{0}{0}\right)$

$$K = \lim_{x\to 0} \frac{x^2+2\cos x-2}{x\left(\dfrac{\sin^3 x}{x^3}\right)\cdot x^3}$$

$$= \lim_{x\to 0} \frac{x^2+2\cos x-2}{x^4} \cdot \lim_{x\to 0}\left(\frac{x}{\sin x}\right)^3$$

$$= \lim_{x\to 0} \frac{2x-2\sin x}{4x^3} \quad \left(\frac{0}{0}\right)$$

$$= \lim_{x\to 0} \frac{2-2\cos x}{12x^2} \quad \left(\frac{0}{0}\right)$$

$$= \lim_{x\to 0} \frac{0+2\sin x}{24x} \quad \left(\frac{0}{0}\right)$$

$$= \lim_{x\to 0} \frac{2\cos x}{24} = \frac{2}{24} = \frac{1}{12}$$

(11) $\lim\limits_{x\to 0} \dfrac{1+\sin x-\cos x+\log(1-x)}{x\tan^2 x} \quad \left(\dfrac{0}{0}\right)$

$$\lim_{x\to 0} \frac{\cos x+\sin x-\dfrac{1}{1-x}}{x\cdot 2\tan x\sec^2 x+\tan^2 x}$$

$$\lim_{x\to 0} \frac{}{x\,\dfrac{2\sin x}{\cos x}\,\dfrac{\sec x}{\cos x}\,\tan^2 x}$$

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(#) $K = \lim_{x \to 0} \frac{1}{x^2} - \frac{1}{\sin^2 x} \quad (\infty - \infty)$

$K = \lim_{x \to 0} \frac{\sin^2 x - x^2}{x^2 \sin^2 x} \rightarrow \left(\frac{0}{0}\right)$

$\lim_{x \to 0} \frac{\sin^2 x - x^2}{x^4} \cdot \lim_{x \to 0} \left[\frac{x}{\sin x}\right]^2$

$\lim_{x \to 0} \frac{\sin^2 x - x^2}{x^4} \cdot 1 \quad \left(\frac{0}{0}\right)$

Applying 'L' Hospitals rule

$K = \lim_{x \to 0} \frac{2\sin x \cos x - 2x}{4x^3} = \lim_{x \to 0} \frac{\sin 2x - 2x}{4x^3} \quad \left(\frac{0}{0}\right)$

$K = \lim_{x \to 0} \frac{2\cos 2x - 2}{12x^2}$ (or) $\lim_{x \to 0} \frac{\cos 2x - 1}{6x^2}$

$= \frac{1}{6} \lim_{x \to 0} \frac{-2\sin^2 x}{x^2}$

$= \frac{-1}{3} \lim_{x \to 0} \left(\frac{\sin x}{x}\right)$

$= \frac{-1}{3} \cdot 1$

$\boxed{K = -1/3}$

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① $K = \lim_{x \to 1} x^{1/(1-x)}$ (1^∞)

$\log K = \lim_{x \to 1} \frac{1}{1-x} \log x$ $(\infty \times 0)$

$= \lim_{x \to 1} \frac{-1}{(1-x)^2} \log x + \frac{1}{x}\left(\frac{1}{1-x}\right)$

$= \lim_{x \to 1} \frac{\log x}{1-x}$

$= \lim_{x \to 1} \frac{1/x}{-1} = \lim_{x \to 1} -\frac{1}{x} = -1$

$\log K = -1 \Rightarrow K = e^{-1}$

$l = -1$

② $K = \lim_{x \to \pi/2} (\sin x)^{\tan x}$

$\log K = \lim_{x \to \pi/2} \tan x \log \sin x$ $(\infty \times 0)$

$= \lim_{x \to \pi/2} \frac{\log \sin x}{\cot x}$ $\left(\frac{0}{0}\right)$

$= \lim_{x \to \pi/2} \frac{1/\sin x \cdot \cos x}{-\operatorname{cosec}^2 x}$

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$$= \lim_{x \to \pi/2} \frac{\cot x}{-\operatorname{cosec}^2 x} = 0$$

$\log K = 0$, $K = e^0 = 1$

(3) $K = \lim_{x \to a} \left[2 - (x/a)\right]^{\tan(\pi x/2a)}$

$$\log K = \lim_{x \to a} \tan(\pi x/2a) \log\left[2 - (x/a)\right] \quad (\infty \times 0)$$

$$= \lim_{x \to a} \frac{\log(2 - x/a)}{\cot\left(\frac{\pi x}{2a}\right)} \quad \left(\frac{0}{0}\right)$$

$$= \lim_{x \to a} \frac{\frac{1}{2 - \frac{x}{a}} \cdot \left(\frac{-1}{a}\right)}{-\operatorname{cosec}^2\left(\frac{\pi x}{2a}\right) \cdot \left(\frac{\pi}{2a}\right)}$$

$$= \lim_{x \to a} \frac{\frac{a}{2a - x} \times \frac{-1}{a}}{-\operatorname{cosec}^2\left(\frac{\pi x}{2a}\right)} \left(\frac{2a}{\pi}\right)$$

$$= \lim_{x \to a} \frac{\frac{\cancel{-}2a}{\pi(2a - x)}}{\cancel{-}\operatorname{cosec}^2\left(\frac{\pi x}{2a}\right)}$$

$$= \lim_{x \to a} \frac{\frac{2a}{\pi(a)}}{\operatorname{cosec}^2\left(\frac{\pi}{2}\right)} = \frac{2}{\pi}$$

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(4) $\lim_{x\to 1} (1-x^2)^{1/\log(1-x)}$

$K = \lim_{x\to 1} (1-x^2)^{\frac{1}{\log(1-x)}}$

$\log K = \frac{1}{\log(1-x)} \log(1-x^2) = \frac{\log(1-x^2)}{\log(1-x)}$

~~$(1-x^2)^{1/\log(1-x)}$~~

$= \frac{\frac{1}{1-x^2} \cdot 2x}{\frac{-1}{1-x}}$

$= \frac{\frac{2x}{1-x^2}}{\frac{-1}{1-x}}$

$= \frac{2x}{1-x^2} \times \frac{1-x}{-1}$

$= \frac{2x(1-x)}{(1-x^2)}$

$= \frac{2x(1-x)}{(1-x^2)}$

$= \frac{2x\cancel{(1-x)}}{(1+x)\cancel{(1-x)}} = \frac{2\times 1}{1+1} = \frac{\cancel{2}}{\cancel{2}} = 1$

$\log K = 1$

$K = e$

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(5) $\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^{1/x^2}$

$\log K = \lim_{x\to 0} \frac{\log\left(\frac{\sin x}{x}\right)}{x^2} = \left(\frac{\infty}{0}\right)$

$= \lim_{x\to 0} \frac{\frac{1}{\left(\frac{\sin x}{x}\right)}\left(\frac{x\cos x - \sin x}{x^2}\right)}{2x}$

$= \lim_{x\to 0} \frac{x}{\sin x} \cdot \lim_{x\to 0} \frac{x\cos x - \sin x}{2x^3} \left(\frac{0}{0}\right)$

$= \lim_{x\to 0} \frac{x(-\sin x) + \cancel{\cos x} - \cancel{\cos x}}{6x^2}$

$= \frac{-1}{6} \lim_{x\to 0} \frac{x\sin x}{x^2} = \frac{-1}{6} \lim_{x\to 0} \frac{x\cos x - \cos x}{2x} \left(\frac{0}{0}\right)$

$= \frac{-1}{6} \lim_{x\to 0} \frac{x(-\sin x) + \cos x + \cos x}{2}$

$= \frac{-1}{6} \frac{\cancel{2}}{\cancel{2}} = \frac{-1}{6}$

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Partial differentiation

(1) If $u = x^3 - 3xy^2 + x + e^x \cos y + 1$

S.T $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

$$\frac{\partial u}{\partial x} = 3x^2 - 3y^2 + 1 + e^x \cos y + 0$$

$$\frac{\partial^2 u}{\partial x^2} = 6x - 0 + 0 + e^x \cos y$$

$$= 6x + e^x \cos y$$

$$\frac{\partial u}{\partial y} = 0 - 6xy + 0 - e^x \sin y + 0$$

$$= -6xy - e^x \sin y$$

$$\frac{\partial^2 u}{\partial y^2} = -6x - e^x \cos y$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 6x + e^x \cos y - 6x - e^x \cos y$$

$$= 0$$

(2) $u = e^{-2\pi^2 t} \sin \pi x \sin \pi y$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{\partial u}{\partial t}$$

$$\frac{\partial u}{\partial x} = e^{-2\pi^2 t} (\pi \cos \pi x) \sin \pi y$$

$$\frac{\partial^2 u}{\partial x^2} = e^{-2\pi^2 t} (-\pi^2 \sin \pi x) \sin \pi y$$

$$= -\pi^2 u$$

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$$\frac{\partial u}{\partial y} = e^{-2\pi^2 t} \sin\pi x \,(\pi\cos\pi y)$$

$$\frac{\partial^2 u}{\partial y^2} = e^{-2\pi^2 t} \sin\pi x \,(-\pi^2 \sin\pi y)$$

$$= -\pi^2 u$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = -\pi^2 u - \pi^2 u$$

$$= -2\pi^2 u$$

$$\frac{\partial u}{\partial t} = e^{-2\pi^2 t} (-2\pi^2) \sin\pi x \sin\pi y$$

$$= -2\pi^2 u$$

$\frac{\partial u}{\partial x} = u_x$ $\quad \frac{\partial^2 u}{\partial x^2} = u_{xx}$

$\frac{\partial u}{\partial y} = u_y$ $\quad \frac{\partial^2 u}{\partial y^2} = u_{yy}$

LHS = RHS

$\frac{\partial}{\partial u}\left(\frac{\partial u}{\partial y}\right) = u_{xy}$

$\frac{\partial}{\partial u}\left(\frac{\partial u}{\partial x}\right) = u_{yx}$

Q3 $u = \log\left(\frac{x^2+y^2}{x+y}\right)$

$x u_x + y u_y = 1$

$u = \log(x^2+y^2) - \log(x+y)$

$$\frac{\partial u}{\partial x} = \frac{1}{x^2+y^2} \cdot 2x - \frac{1}{2+y} = u_x$$

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$$\frac{2x^2}{x^2+y^2} - \frac{x}{x+y} = u_x u_x$$

$$= \frac{2x}{x^2+y^2} - \frac{1}{x+y}$$

$$\frac{\partial u}{\partial y} = \frac{1}{x^2+y^2} 2y - \frac{1}{x+y} u_y = \frac{2y^2}{x^2+y^2} - \frac{y}{x+y} = y u_y$$

$$= \frac{2y}{x^2+y^2} - \frac{1}{x+y}$$

$$x u_x + y u_y = 1$$

$$\frac{2x^2}{x^2+y^2} - \frac{x}{x+y} + \frac{2y^2}{x^2+y^2} - \frac{y}{x+y} = 1$$

$$\left(\frac{2x^2}{x^2+y^2} + \frac{2y^2}{x^2+y^2}\right) \left(\frac{x}{x+y} + \frac{y}{x+y}\right) = 1$$

$$= \frac{2(x^2+y^2)}{x^2+y^2} - \frac{x+y}{x+y} = 1$$

$$\boxed{2-1=1}$$

LHS = RHS

(4) If $z = \tan(y+ax) + (y-ax)^{3/2}$

S.T $\frac{\partial^2 z}{\partial x^2} - a^2 \frac{\partial^2 z}{\partial y^2} = 0$

$$\frac{\partial z}{\partial x} = \sec^2 x(y+ax.\ a + 3/2\,(y-ax)^{3/2}(-a)$$

diff

$$\frac{\partial^2 z}{\partial x^2} = 2\sec(y+ax).\tan(y+ax) + 3/4\,(y-ax)^{-1/2}\cdot$$

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$$\frac{\partial z}{\partial y} = \sec^2(y+ax) + \frac{3}{2}(y-ax)^{-1/2}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) = 2\sec(y+ax)\sec(y+ax).\tan(y+ax) + \frac{3}{2}\cdot\frac{1}{2}(y-ax)^{-1/2}$$

$$\frac{\partial^2 z}{\partial y^2} = 2\sec^2(y+ax)\tan(y+ax + \frac{3}{4}(y-ax)^{-1/2}$$

$$\frac{\partial^2 z}{\partial x^2} = a.\frac{\partial^2 z}{dy^2}$$

$$\frac{\partial^2 z}{\partial x^2} - a.\frac{\partial^2 z}{\partial y^2} = 0$$

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(1) If $u = \log\sqrt{x^2+y^2+z^2}$ then S.T $(x^2+y^2+z^2)\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) = 1$

$$u = \frac{1}{2}\log(x^2+y^2+z^2)$$

$$\frac{\partial u}{\partial x} = \frac{1}{2}\frac{2x}{(x^2+y^2+z^2)}$$

$$\frac{\partial u}{\partial x} = \frac{x}{x^2+y^2+z^2}$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{(x^2+y^2+z^2)(1) - x(2x)}{(x^2+y^2+z^2)}$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{(x^2+y^2+z^2) - 2x^2}{(x^2+y^2+z^2)^2} = \frac{y^2+z^2-x^2}{(x^2+y^2+z^2)^2}$$

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$$\frac{\partial^2 u}{\partial y^2} = \frac{z^2+x^2-y^2}{(x^2+y^2+z^2)^2}$$

$$\frac{\partial^2 u}{\partial z^2} = \frac{x^2+y^2-z^2}{(x^2+y^2+z^2)^2}$$

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2} = \frac{\cancel{y^2}+\cancel{z^2}-\cancel{x^2}+\cancel{z^2}+\cancel{x^2}-\cancel{y^2}+x^2+y^2-\cancel{z^2}}{(x^2+y^2+z^2)^2}$$

$$= \frac{x^2+y^2+z^2}{(x^2+y^2+z^2)^2} = \frac{1}{(x^2+y^2+z^2)}$$

$$(x^2+y^2+z^2)\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right) = \cancel{x^2+y^2+z^2}\left(\frac{1}{\cancel{x^2+y^2+z^2}}\right)$$

$$= 1$$

(2) $u = \dfrac{1}{\sqrt{x^2+y^2+z^2}}$ S.T $\left(\dfrac{\partial^2 u}{\partial x^2}+\dfrac{\partial^2 u}{\partial y^2}+\dfrac{\partial^2 u}{\partial x^2}\right) = 0$

$$u = (x^2+y^2+z^2)^{-1/2}$$

$$\frac{\partial u}{\partial x} = \frac{-1}{\cancel{2}}(x^2+y^2+z^2)^{-3/2}\cdot \cancel{2}x$$

$$= -x(x^2+y^2+z^2)^{-3/2}$$

$$\frac{\partial^2 u}{\partial x^2} = -\left[x\left(\frac{-3}{2}(x^2+y^2+z^2)^{-5/2}\cdot 2x + (x^2+y^2+z^2)^{-3/2}(1)\right)\right]$$

$$\frac{\partial^2 u}{\partial x^2} = 3x^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2}$$

$$\frac{\partial^2 u}{\partial x^2} = 3y^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2}$$

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$$\frac{\partial^2 u}{\partial z^2} = 3z^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2}$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = (3x^2+3y^2+3z^2)(x^2+y^2+z^2)^{-5/2}$$
$$-3(x^2+y^2+z^2)^{-3/2}$$

$$= 3\left[(x^2+y^2+z^2)(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2}\right]$$

$$= 3\left[\cancel{(x^2+y^2+z^2)^{-3/2}} - \cancel{(x^2+y^2+z^2)^{-3/2}}\right] = 0$$

③ $u = \log(x^3+y^3+z^3-3xyz)$

S.T $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x+y+z}$

P.T $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x+y+z)^2}$

$$\frac{\partial u}{\partial x} = \frac{1}{x^3+y^3+z^3-3xyz} \cdot 3x^2 - 3yz$$

$$\frac{\partial u}{\partial x} = \frac{3x^2-3yz}{x^3+y^3+z^3-3xyz}$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{(x^3+y^3+z^3-3xyz)(6x) - (3x^2-3yz)(3x^2-3yz)}{(x^3+y^3+z^3-3xyz)^2}$$

$$\frac{\partial u}{\partial y} = \frac{3y^2-3zx}{y^3+x^3+z^3-3xyz}$$

$$\frac{\partial u}{\partial z} = \frac{3z^2-3xy}{x^3+y^3+z^3-3xyz}$$

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Adding

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{z}=\frac{3(x^2+y^2+z^2-xy-yz-zx)}{x^3+y^3+z^3-3xyz}$$

$$=\frac{3(x^2+y^2+z^2-yz-zx-xy)}{(x^2+yz+z^3-3xyz)}$$

$$=\frac{3\cancel{(x+y+z)^2}}{(x+y+z)^{\cancel{3}}}$$

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=\frac{3}{(x+y+z)}$$

P.T $\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^2 u=\frac{-9}{(x+y+z)^2}$

$$\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)u$$

$$\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)\left(\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}\right)$$

$$\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)=\left(\frac{3}{x+y+z}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{3}{x+y+z}\right)+\frac{\partial}{\partial y}\left(\frac{3}{x+y+z}\right)+\frac{\partial}{\partial z}\left(\frac{3}{x+y+z}\right)$$

$$\frac{-3}{(x+y+z)^2}-\frac{3}{(x+y+z)^2}-\frac{3}{(x+y+z)^2}=\frac{-9}{(x+y+z)^2}$$

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Find the minimum Value of Lagrange's method of multipliers

(1) $x^2+y^2+z^2$ when $x+y+z=3a$

$F=x^2+y^2+z^2$

$\phi=x+y+z=3a$

$F+\lambda(\phi)$

$=(x^2+y^2+z^2)+\lambda(x+y+z)$

$F_x = 2x+\lambda \Rightarrow 2x+\lambda=0 \Rightarrow \lambda=-2x$

$F_y = 2y+\lambda \Rightarrow 2y+\lambda=0 \Rightarrow \lambda=-2y$

$F_z = 2z+\lambda \Rightarrow 2z+\lambda=0 \Rightarrow \lambda=-2z$

$-2x = -2y = -2z$

$\cancel{-}\cancel{2}x = \cancel{-}\cancel{2}y$

$\boxed{x=y}$ $\Rightarrow x=y=z$

$y=z, \quad x=z$

$x=y$

$F = x^2+x^2+x^2$

$F=3x^2$

$x+y+z=3a$

$x+x+x=3a$

$\cancel{3}x=\cancel{3}a$

$\boxed{x=a}$

$y+y+y=3a$

$\cancel{3}y=\cancel{3}a$

$\boxed{y=a}$

$F=x^2+y^2+z^2$

$=a^2+a^2+a^2$

$z+z+z=3a$

$\boxed{z=a}$

$\boxed{F=3a^2}$

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(2) Find the minimum value of $x^2+y^2+z^2$ subject to the condition $xy+yz+zx=3a^2$

$F = x^2+y^2+z^2$
$\phi = xy+yz+zx = 3a^2$

$F+\lambda(\phi)$
$x^2+y^2+z^2+\lambda(xy+yz+zx)$
$F_x = 2x+\lambda(y+z)$
$F_y = 2y+\lambda(x+z)$
$F_z = 2z+\lambda(y+x)$

$F_x = F_y = F_z = 0$
$\lambda = \frac{-2x}{y+z}, \lambda = \frac{-2y}{(x+z)}, \lambda = \frac{-2z}{(y+x)}$

$\frac{-2x}{y+z} = \frac{-2y}{x+z} = \frac{-2z}{y+x}$

$\frac{\cancel{-2}x}{y+z} = \frac{\cancel{-2}y}{x+z}$
$x(x+z) = y(y+z)$
$x^2+zx = y^2+yz$
$x^2-y^2+xz-yz = 0$
$x^2-y^2+z(x-y) = 0$
$(x+y)(x-y)+z(x-y) = 0$
$(x-y)(x+y+z) = 0$
$x-y=0, \quad x+y+z=0$
$x=y, \quad x+y+z=0$

Case i) $x=y, \ y=z, \ z=x$
Case ii) $x+y+z=0$

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from case ii)

$$(x+y+z)^2 = 0$$
$$x^2+y^2+z^2+xy+yz+zx = 0$$
$$x^2+y^2+z^2+3x^2 = 0$$
$$F = -3a^2 < 0$$

1) $x = y = z$

$$xy+yz+zx = 3a^2$$
$$x^2+x^2+x^2 = 3a^2$$
$$\cancel{3}x^2 = \cancel{3}a^2$$
$$x^2 = a^2,\ y^2 = a^2,\ z^2 = a^2$$

$$F = a^2+a^2+a^2$$
$$= 3a^2$$

(3) Find the stationary value of x^2, y^3, z^4 subject to the condition $x+y+z = 5$

$$F = x^2 * y^3 * z^4$$
$$\phi = x+y+z = 5$$

$$f + \lambda(\phi)$$
$$x^2+y^3+z^4+\lambda\cancel{(x*y*z)}(x+y+z)$$
$$\cancel{F_x = 2x+\lambda(y+z)}\quad f_x = 2xy^3z^4+\lambda = 0$$
$$\cancel{F_y = 3y^2+\lambda(x+z)}\quad f_y = 3x^2y^2z^4+\lambda = 0$$
$$\cancel{F_z = 4z^3+\lambda}\quad f_z = 4x^2y^3z^3+\lambda = 0$$

$$f_x = \lambda = -2xy^3z^4$$
$$f_y = \lambda = -3x^2y^2z^4$$
$$f_z = \lambda = -4x^2y^3z^3$$

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$f_x = f_y = f_z$

(1) $-2xy^3z^4 = -3x^2y^2z^4 = -4x^2y^3z^3$

$\cancel{-}2\cancel{x}\cancel{y^3}\cancel{z^4} = \cancel{-}3\cancel{x^2}\cancel{y^2}\cancel{z^4}$

$2y = 3x$

(2) $\cancel{-}3\cancel{x^2}\cancel{y^2}z^{\cancel{4}} = \cancel{-}4\cancel{x^2}y^{\cancel{3}}\cancel{z^3}$

$3z = 4y$

(3) $\cancel{-}\cancel{4}x^2\cancel{y^3}\cancel{z^3} = \cancel{2}\cancel{x}\cancel{y^3}z^4$

$4x = 2z$

$2y = 3x, \quad 3z = 4y, \quad \cancel{4}x = 2z$

$y = 3x/2, \quad z = 2x,$

$x + y + z = 5$

$x + (3x/2) + 2x = 5$

$\boxed{x = \frac{10}{9}}$

$x + \frac{3x}{2} + 2x = 5$

$3x + \frac{3x}{2} = 5$

$6 + \frac{9x}{2} = 5$

$9x = 10$

$x = \frac{10}{9}$

$y = \frac{3}{2}x$

$y = \frac{3}{2} \times \frac{10}{9}$

$= \frac{30}{18}$

$\boxed{y = \frac{5}{3}}$

$z = 2x$

$z = 2 \times \frac{10}{9}$

$\boxed{z = \frac{20}{9}}$

1.234×4.629

$\times 24.386$

$F = x^2 y^3 z^4$

$= \left(\frac{10}{9}\right)^2 \left(\frac{5}{3}\right)^3 \left(\frac{20}{9}\right)^4$

$\frac{100}{81} \times \frac{125}{27} \times \frac{160000}{6561}$

$= \frac{\cancel{100}}{\cancel{81}}$

(#) Find the extreme values of xy subject to the condition $x^2+xy+y^2=a^2$

Sol:- $F = xy$

$\phi = x^2+xy+y^2$

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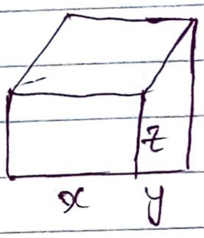
Applications of maxima & Minima

A rectangular box open at the top is to have a volume of 32 cubic feet. Find its dimension if the total surface area is minimum.

Sol:- $x, y, z \longrightarrow l, b, h$

Total surface =



$$T.S = 2(xy + yz + zx)$$
$$S = 2xy + 2yz + 2zx - xy$$
$$S = xy + 2yz + 2zx$$
$$V = xyz$$

$$F + \lambda\phi = (xy + 2yz + 2zx) + \lambda(xyz)$$
$$F_x = F_y = F_z = 0$$
$$F_x = (y + 2z) + \lambda(yz) \Rightarrow \lambda = -(y + 2z)/yz$$
$$F_y = (x + 2z) + \lambda(xz) \Rightarrow \lambda = -(x + 2z)/xz$$
$$F_z = (2y + 2x) + \lambda(xy) \Rightarrow \lambda = -(2y + 2x)/xy$$

$$-\frac{(y + 2z)}{yz} = -\frac{(x + 2z)}{xz} = -\frac{(2y + 2x)}{xy}$$

$$\frac{\cancel{-}(y + 2z)}{y\cancel{z}} = \frac{\cancel{-}(x + 2z)}{x\cancel{z}} \Rightarrow x(y + 2z) = y(x + 2z)$$
$$\cancel{xy} + 2xz = \cancel{yx} + 2yz$$

$$\boxed{x = y}$$

$$\frac{\cancel{+}(x+2z)}{xz} = \cancel{+}\frac{(2y+2x)}{\cancel{x}y}$$

$$yx + 2yz = 2yz + 2xz$$

$$\boxed{y = 2z}$$

$$x = y = 2z$$

$$x = 2z \Rightarrow \boxed{z = x/2}$$

$$V = (x)(x)(x/2)$$

$$32 = \frac{x^3}{2} \Rightarrow x^3 = 64 \Rightarrow \boxed{x = 4}$$

$$x = y = 4, \quad z = \frac{x}{2} = \frac{4}{2} = 2$$

$$(x, y, z) = (4, 4, 2)$$

(#) Find the volume of largest rectangular parallelopiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

$$V = xyz$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

$$F + \lambda\phi$$

$$xyz + \lambda\left(\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2}\right) \Rightarrow \cancel{\lambda = \frac{yz}{2}}$$

$$F_x = yz + \lambda\left(\frac{2x}{a^2}\right) \Rightarrow \lambda = -\frac{yz\,a^2}{2x}$$

$$F_y = zx + \lambda\left(\frac{2y}{b^2}\right) \Rightarrow \lambda = -\frac{zx\,b^2}{2y}$$

$$F_z = xy + \lambda\left(\frac{2z}{c^2}\right) \Rightarrow \lambda = -\frac{xy\,c^2}{2z}$$

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$$-\frac{yza^2}{2x} = -\frac{zxb^2}{2y} = -\frac{xyc^2}{2z}$$

$$+\frac{yza^2}{2x} = +\frac{zxb^2}{2y}$$

$$y(ya^2) = x(xb^2)$$

1) $y^2a^2 = x^2b^2$ 2) $z^2b^2 = y^2c^2$ 3) $x^2c^2 = z^2a^2$

$$y^2 = \frac{x^2b^2}{a^2} \qquad z^2 = \frac{x^2c^2}{a^2}$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \Rightarrow \frac{x^2}{a^2} + \frac{\frac{x^2b^2}{a^2}}{b^2} + \frac{\frac{x^2c^2}{a^2}}{c^2} = 1$$

$$= \frac{x^2}{a^2} + \frac{x^2}{a^2} + \frac{x^2}{a^2} \Rightarrow \frac{3x^2}{a^2} = 1 \Rightarrow 3x^2 = a^2 \Rightarrow x = \frac{a}{\sqrt{3}}$$

$$y = b/\sqrt{3}, \quad z = c/\sqrt{3}$$

$$V = xyz = \frac{a}{\sqrt{3}}\,\frac{b}{\sqrt{3}}\,\frac{c}{\sqrt{3}} = \frac{abc}{3\sqrt{3}}$$

<u>Jacobian</u>

91%

80%

$$J = \frac{\partial(u,v,w)}{\partial(x,y,z)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial u}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{vmatrix}$$

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Find the Jacobian of u, v, w w.r.t x, y, z given $u = x+y+z$, $v = y+z$, $w = z$

$$J = \begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{vmatrix} \Rightarrow \boxed{J=1}$$

Find J where
$u = x^2+y^2+z^2$, $v = xy+yz+zx$, $w = x+y+z$

$$J = \begin{vmatrix} 2x & 2y & 2z \\ y+z & x+z & y+x \\ 1 & 1 & 1 \end{vmatrix}$$

$$2x[(x+z)-(y+x)] - 2y[(y+z)-(y+x)] + 2z[(y+z)-(x+z)]$$

$$= 2x(z-y) - 2y(z-x) + 2z(y-x)$$
$$= 2[\cancel{xz} - \cancel{xy} - \cancel{yz} + \cancel{yx} + \cancel{zy} - \cancel{zx}]$$
$$= \underline{0}$$

If $x+y+z = u$, $y+z = v$ & $z = u, v, w$
find $\dfrac{\partial(x,y,z)}{\partial(u,v,w)}$

$x + v = u$ $\qquad y + uv +$
$x = u - v$ $\qquad y = v - uv$

$z = uvw$