17/12/18

SURYA Gold

Date ______ Page ______

Module - 4
Separation of Variables

① Solve

$$xy\frac{dy}{dx} = 1 + x + y + xy$$

$$= (1+x) + y(1+x)$$

$$xy\frac{dy}{dx} = (1+x)(1+y)$$

$$\frac{y\,dy}{1+y} = \frac{(1+x)}{x}dx$$

$$\int \frac{y}{1+y}dy = \int \frac{1+x}{x}dx$$

$$\int \frac{(y+1)-1}{1+y} - \int \frac{1+x}{x}dx = c$$

$$\int dy - \int \frac{1}{y}dy - \int \frac{1}{x}dx - \int dx = c$$

$$y - \log y - \log x - x = c$$

$$y - x - \log\left(\frac{y}{x}\right) = c$$

19/12/18

SURYA Gold

Date ____ Page ____

Exact Equation

(#) Solve $(2x+y+1)dx + (x+2y+1)dy = 0$

$M = 2x+y+1$, $N = x+2y+1$

$\frac{\partial M}{\partial y} = 0+1 = 1$, $\frac{\partial N}{\partial x} = 1+0+0 = 1$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \longrightarrow$ Exact solⁿ

$\int M\,dx + \int N(y)\,dy = C$

$\int (2x+y+1)\,dx + \int (2y+1)\,dy = C$

$\left(\cancel{2}\frac{x^2}{\cancel{2}} + xy + x\right) + \left(\cancel{2}\frac{y^2}{\cancel{2}} + y\right) = C$

$x^2 + xy + x + y^2 + y = C$

(#) Solve $(5x^4 + 3x^2y^2 - 2xy^3)dx + (2x^3y - 3x^2y^2 - 5y^4)dy = 0$

$M = 5x^4 + 3x^2y^2 - 2xy^3$, $N = 2x^3y - 3x^2y^2 - 5y^4$

$\frac{\partial M}{\partial y} = 0 + 6x^2y - 6xy^2$, $\frac{\partial N}{\partial x} = 6x^2y - 6xy^2 - 0$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \longrightarrow$ Exact

$\int M\,dx + \int N(y)\,dy = C$

$\int (5x^4 + 3x^2y^2 - 2xy^3)\,dx + \int -5y^4\,dy = C$

$\frac{\cancel{5}x^5}{\cancel{5}} + \frac{\cancel{3}x^3}{\cancel{3}}y^2 - \frac{\cancel{2}x^2}{\cancel{2}}y^3 - \frac{\cancel{5}y^5}{\cancel{5}} = C$

$$x^5 + x^3y^2 - x^2y^3 - y^5 = C$$

\# Solve $\dfrac{dy}{dx} + \dfrac{(x+3y-4)}{3x+9y-2} = 0$

$$\frac{dy}{dx} = \frac{(x+3y-4)}{(3x+9y-2)}$$

$$(3x+9y-2)\,dy + (x+3y-4)\,dx$$

$$M = (x+3y-4), \quad N = (3x+9y-2)$$

$$\frac{\partial M}{\partial y} = 0+3 = 0 \qquad \frac{\partial N}{\partial x} = 3+0+0 = 3$$
$$= 3$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$\int M\,dx + \int N(y)\,dy = C$$

$$\int (x+3y-4)\,dx + \int (9y-2)\,dy = C$$

$$\frac{x^2}{2} + 3xy - 4x + \frac{9y^2}{2} - 2y = C$$

$$\frac{x^2}{2} + 3xy - 4x + \frac{9}{2}y^2 - 2y = C$$

$$\frac{x^2}{2} + 3xy - 4x + \frac{9}{2}y^2 - 2y = C$$

$$\frac{1}{2}\left[x^2 + \frac{3}{2}xy - 2x + 9y^2 - y\right) = C$$

$$x^2 +$$

Solve $\frac{dy}{dx} + \frac{y\cos x + \sin y + y}{\sin x + x\cos y + x} = 0$

$$\frac{dy}{dx} = \frac{-(y\cos x + \sin y + y)}{\sin x + x\cos y + x}$$

$$(y\cos x + \sin y + y)\,dx + (\sin x + x\cos y + x)\,dy = 0$$

$$M = y\cos x + \sin y + y, \quad N = \sin x + x\cos y + x$$

$$\frac{\partial M}{\partial y} = \cos x + \cos y + 1, \quad \frac{\partial N}{\partial x} = \cos x + \cos y + 1$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \rightarrow \text{Exact sol}^n$$

$$\int (y\cos x + \sin y + y)\,dx + \int 0\,dy = c$$

$$y\sin x + \sin y\, x + yx = c$$

$$y\sin x + x\sin y + xy = c$$

20/12/18

Solve $(y^2 e^{xy^2} + 4x^3)\,dx + (2xye^{xy^2} - 3y^2)\,dy = 0$

$$M = (y^2 e^{xy^2} + 4x^3), \quad N = 2xye^{xy^2} - 3y^2$$

$$\frac{\partial M}{\partial y} = y^2 e^{xy^2} 2xy + e^{xy^2} 2y, \quad \frac{\partial N}{\partial x} = 2xye^{xy^2} \cdot y^2 + e^{xy^2} \cdot 2y - 0$$

$$= \int M\,dx + \int N(y)\,dy = c$$

$$= \int (y^2 e^{xy^2} + 4x^3)\,dx + \int -3y^2\,dy = c$$

20/12/18

$$\cancel{y^2}\frac{e^{xy^2}}{\cancel{y^2}} + \cancel{4}\frac{x^4}{\cancel{4}} - \cancel{3}\frac{y^3}{\cancel{3}} = c$$

$$e^{xy^2} + x^4 - y^3 = c$$

Equations reducable to exact form

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

$$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}$$

$$\frac{1}{M}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) \to g(y) \quad \text{or} \quad \frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = f(x)$$

$$e^{\int f(x)\,dx} \qquad e^{-\int g(y)\,dy}$$

$$\int M\,dx + \int N(y)\,dy$$

$$e^{\log x} = x, \quad e^{n\log x} = x^n$$

\# Solve $(4xy + 3y^2 - x)\,dx + x(x+2y)\,dy = 0$

$M = 4xy + 3y^2 - x$, $\quad N = x^2 + 2xy$

$$\frac{\partial M}{\partial y} = 4x + 6y = 2(2x+3y), \quad \frac{\partial N}{\partial x} = 2x + 2y = 2(x+y)$$

$$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = 4x + 6y - 2x - 2y$$

$$= 2x + 4y$$

$$= 2(x+2y)$$

SURYA Gold

Date______ Page______

$$\frac{1}{N}\left(\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}\right)=\frac{2\cancel{(x+2y)}}{x\cancel{(x+2y)}}$$

$$=\frac{2}{x}=f(x)$$

$$e^{\int f(x)\,dx}=e^{\int \frac{2}{x}dx}=e^{2\log x}=e^{\log x^2}=x^2$$

$$M=x^2(4xy+3y^2-x),\quad N=x^2(x^2+2xy)$$

$$=4x^3y+3x^2y^2-x^3,\quad N=x^4+2x^3y$$

$$\frac{\partial M}{\partial y}=4x^3+6x^2y-0,\quad \frac{\partial N}{\partial x}=4x^3+6x^2y$$

$$\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$$

$$\int M\,dx+\int N(y)\,dy=c$$

$$\int(4x^3y+3x^2y^2-x^3)\,dx+\int 0\,dy=c$$

$$\frac{\cancel{4}x^4y}{\cancel{4}}+\frac{\cancel{3}x^3}{\cancel{3}}-\frac{x^4}{4}=c,$$

$$x^4y+x^3y^2-\frac{x^4}{4}=c$$

\# Solve $y(2x-y+1)\,dx+x(3x-4y+3)\,dy=0$

$M=y(2x-y+1)$ $\qquad N=3x^2-4y+3$

$\frac{\partial M}{\partial y}=2x-2y+1$ $\qquad \frac{\partial N}{\partial x}=6x+4y+3$

$\cancel{2(1-y)}$

$$\frac{\partial M}{\partial y}\neq\frac{\partial N}{\partial x}\quad =$$

$2x - 2y + 1 - \cancel{6x - 4y + 3}$

$\cancel{= -4x + 2y + 4}$

$= -4x + 2y - 2$

$= 2(-2x + y - 1)$

$\cancel{= \frac{-2}{y}} = g(y)$

$$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = 2x - 2y + 1 \,,\; -6x + 4y - 3$$

$$= -4x + 2y - 2$$

$$\frac{1}{M}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = \frac{2(2x - y + 1)}{2xy - y^2 + y}$$

$$= -\frac{2(2x - y + 1)}{}$$

(#) Solve $y(x+y)\,dx + (x+2y-1)\,dy = 0$

$M = y(x+y)$ $\qquad$ $N = (x+2y-1)$

$xy + y^2$

$\frac{\partial M}{\partial y} = x + 2y$ $\qquad$ $\frac{\partial N}{\partial x} = x + 2y - 1$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

$$\frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = \frac{\cancel{x+2y-1}}{\cancel{x+2y-1}} = 1$$

$$\Rightarrow f(x) = 1$$

$$e^{\int f(x)\,dx} = e^{\int \frac{2}{x}dx} = e^{2\log x} = e^{\log x^2} = x^2 \Rightarrow e^x$$

$$= e^x$$

$M = e^x(xy + y^2)$, $N = e^x(x + 2y - 1)$

$e^x x + e^x \cdot 2y$ $\qquad$ $= e^x(x + 2y - 1)\,e^x$

$\frac{\partial M}{\partial y} = xe^x + 2e^x y$ and $\frac{\partial N}{\partial x} = xe^x + 2e^x y$

$$\int M\,dx + \int N(y)\,dy = C$$

$$y\int xe^x\,dx + y^2\int e^x\,dx + \int 0\,dy = C$$

$$y(xe^x - e^x) + y^2 e^x = C$$

$$\therefore e^x(xy + y^2 - y) = C$$

SURYA Gold

Date ______ Page ______

Bernoulli's Differential equation

$$\frac{dy}{dx} + Py = Q$$

$$ye^{\int P dx} = \int Q e^{\int P dx} dx + C$$

$$\frac{dx}{dy} + Px = Q$$

$$xe^{\int P dy} = \int Q e^{\int P dy} dy + C$$

26/12/18

(1) $x^{2/3} + y^{2/3} = a^{2/3}$

$$\frac{2}{3} x^{-1/3} + \frac{2}{3} y^{-1/3} \frac{dy}{dx} = 0$$

$$\frac{2}{3}\left(x^{-1/3} + y^{-1/3} \frac{dy}{dx}\right) = 0$$

$$\frac{dy}{dx} \longrightarrow -\frac{dx}{dy}$$

$$\frac{2}{3}\left(x^{-1/3} + y^{-1/3}\left(\frac{-dx}{dy}\right)\right) = 0$$

$$\frac{2}{3} x^{-1/3} = \frac{2}{3} y^{-1/3} \frac{dx}{dy}$$

$$\frac{1}{x^{1/3}} = \frac{1}{y^{1/3}} \frac{dx}{dy}$$

$$y^{1/3} dy = x^{1/3} dx$$

$$\int x^{1/3} dx - \int y^{1/3} dy = c$$

$$\frac{x^{4/3}}{4/3} - \frac{y^{4/3}}{4/3} = c$$

$$\frac{3}{4}\left(x^{4/3} - y^{4/3}\right) = c$$

$$3\left(x^{4/3} - y^{4/3}\right) = 4c = k \text{ (say)}$$

(2) Find the orthogonal trajectories of family $r = a(1+\sin\theta)$

$$\log r = \log a + \log(1+\sin\theta)$$

$$\frac{1}{r}\frac{dr}{d\theta} = 0 + \frac{\cos\theta}{1+\sin\theta}$$

$$\frac{dr}{d\theta} \longrightarrow -r^2\frac{d\theta}{dr}$$

$$\frac{1}{r}\left(-r^2\frac{d\theta}{dr}\right) = \frac{\cos\theta}{1+\sin\theta}$$

$$-r\frac{dr}{d\theta} = \frac{\cos\theta}{1+\sin\theta}$$

$$-\int r\frac{dr}{d\theta} = \int\frac{\cos\theta}{1+\sin\theta}$$

$$\frac{1+\sin\theta}{\cos\theta}\,d\theta = -\frac{dr}{r}$$

$$(\sec\theta + \tan\theta)\,d\theta = -\frac{dr}{r}$$

$$\int\sec\theta\,d\theta + \int\tan\theta\,d\theta = -\int\frac{dr}{r}$$

$$\log(\sec\theta + \tan\theta) + \log(\sec\theta) = -\log r + c$$

$$\log r + \log(\sec\theta + \tan\theta) + \log(\sec\theta) = c$$

$$\log\left[r(\sec\theta + \tan\theta)\sec\theta\right] = c$$

$$\log\left[r\left(\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}\right)\frac{1}{\cos\theta}\right] = c$$

SURYA Gold

Date ______ Page ______

$$\log\left[r\left(\frac{1}{\cos^2\theta}+\frac{\sin\theta}{\cos^2\theta}\right)\right]=c$$

$$\log\left[r\left(\frac{1+\sin\theta}{\cos^2\theta}\right)\right]=c$$

$$\log\left[r\left(\frac{1+\sin\theta}{1-\sin^2\theta}\right)\right]=c$$

$$\log\left[r\,\frac{1+\sin\theta}{(1+\sin\theta)(1-\sin\theta)}\right]=c$$

$$\log\left[\frac{r}{1-\sin\theta}\right]=c=\log b \text{ (say)}$$

$$\frac{r}{1-\sin\theta}=b \Rightarrow r=b(1-\sin\theta)$$

(13) Find the O.T of the family

$$r^n\cos n\theta=a^n$$

$$\log r^n\cos n\theta=\log a^n$$

$$n\log r+\log(\cos n\theta)=0$$

$$n\cdot\frac{1}{r}\frac{dr}{d\theta}+\frac{1}{\cos n\theta}-\sin n\theta\cdot n$$

$$\cancel{n}\cdot\frac{1}{r}\frac{dr}{d\theta}=-\cancel{n}\left[\frac{-\sin n\theta}{\cos n\theta}\right]$$

$$\frac{1}{r}\frac{dr}{d\theta}=\tan n\theta$$

$$\frac{dr}{d\theta}\rightarrow -r^2\frac{d\theta}{dr}$$

$$\frac{1}{\cancel{r}}\left(-r^{\cancel{2}}\frac{d\theta}{dr}\right)=\tan n\theta$$

$$\Rightarrow -r\frac{d\theta}{dr} = \tan n\theta$$

$$-r\frac{d\theta}{\tan n\theta} = dr$$

$$\frac{d\theta}{\tan n\theta} = \frac{-dr}{r}$$

$$\int \frac{1}{\tan n\theta}\, d\theta = -\int \frac{dr}{r}$$

$$\int \cot n\theta \cdot d\theta = -\log r + c$$

$$\frac{\log|\sin n\theta|}{n} = -\log r + c$$

$$\frac{\log|\sin n\theta|}{n} + \log r = c$$

$$\frac{1}{n}\left(\log|\sin n\theta| + n\log r\right) = c$$

$$\log(\sin n\theta) + \log r^n = nc$$

$$\log(\sin n\theta \cdot r^n) = nc$$

$$\boxed{r^n \sin n\theta = b}$$

(4) S.T the O.T of the family of cardiods $r = \frac{a\cos^2\theta}{2}$ is another family of cardiods

$$r = \frac{b\sin^2\theta}{2}$$

$$r = \frac{a\cos^2\theta}{2}$$

$$\log r = \log a + 2\log \cos(\theta/2)$$

$$\frac{1}{r}\frac{dr}{d\theta} = \cancel{2}\cdot \frac{1}{\cos(\theta/\cancel{2})} - \sin(\theta/2)\cdot 1/\cancel{2}$$

$$\frac{1}{r}\frac{dr}{d\theta} = -\tan\theta/2$$

$$\frac{dr}{d\theta} = -r^2\frac{dr}{d\theta}$$

$$-r\frac{d\theta}{dr} = -\tan\theta/2$$

$$\cot\theta/2\, d\theta = \frac{dr}{r}$$

$$\int \cot\theta/2\, d\theta = \int \frac{dr}{r}$$

$$\frac{\log(\sin\theta/2)}{1/2} = \log r + C$$

$$\cancel{2\log}\ \log r - 2\log\sin(\theta/2) = C$$

$$\log\left(\frac{r}{\sin^2(\theta/2)}\right) = \log b \text{ (say)}$$

$$\boxed{\text{Thus } r = b\sin^2(\theta/2)}$$

27/12/18

SURYA Gold

Date______ Page______

Using the concept of O.T. S.T the family of curves

$$r = a(\sin\theta + \cos\theta) \text{ and } r = b(\sin\theta - \cos\theta)$$

intersect each other orthogonally

$r = a(\sin\theta + \cos\theta)$

$\log r = \log a + \log(\sin\theta + \cos\theta)$

$\frac{1}{r}\frac{dr}{d\theta} = 0 + \frac{\cos\theta - \sin\theta}{\sin\theta + \cos\theta}$

$r = b(\sin\theta - \cos\theta)$

$\log r = \log b + \log(\sin\theta - \cos\theta)$

$\frac{1}{r}\frac{dr}{d\theta} = 0 + \frac{1}{\sin\theta - \cos\theta}(\cos\theta - \sin\theta)$

$\frac{1}{r}\frac{dr}{d\theta} = \frac{\cos\theta - \sin\theta}{\sin\theta - \cos\theta}$

$\frac{dr}{d\theta} \to -r^2\frac{d\theta}{dr}$

$-r\frac{d\theta}{dr} = \frac{\cos\theta - \sin\theta}{\sin\theta + \cos\theta}$

$\frac{\sin\theta + \cos\theta}{\cos\theta - \sin\theta}\, d\theta = -\frac{dr}{r}$

$\frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} = -\frac{dr}{r}$

$\int \frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta}\, d\theta = \frac{dr}{r}$

$\log r - \log|\cos\theta + \sin\theta| = c$

$\log\left(\frac{r}{\cos\theta - \sin\theta}\right) = \log b \text{ (say)}$

$\frac{r}{\cos\theta - \sin\theta} = b$

$r = b(\sin\theta - \cos\theta)$

Newton's law of cooling

$$T = t_2 + (t_1 - t_2)\, e^{-kt}$$

t_1 = initial temperature of the body

t_2 = constant temperature of the body

T = Temperature of the body at anytime (t)

① A body in air at $\overset{t_2}{25^\circ C}$ cools from $\overset{t_1}{100^\circ C}$ to $75^\circ C$ in one minute. Find the temperature of the body at the end of 3 minutes.

Sol:-

$t_1 = 100$

$t_2 = 25$

$T = 75$

$$T = 25 + (100 - 25)\, e^{-kt}$$

$$75 = 25 + (100 - 25)\, e^{-k}$$

$$75 = 25 + 75e^{-k}$$

$$75e^{-k} = 75 - 25$$

$$75e^{-k} = 50$$

$$e^{-k} = \cancel{\frac{75}{50}}\ \frac{50}{75} = \frac{2}{3} \Rightarrow e^{k} = \frac{3}{2} = 1.5$$

$$\Rightarrow k = \log(1.5) = 0.4055$$

$$(T)_{t=3} = 25 + 75e^{-0.4055(3)}$$

$$= 25 + 75e^{-1.2165}$$

$$= 47.2198 \simeq \underline{47.22}$$

(2) If the temperature of the air is 30°C and the metal ball cools from 100° to 70°C in 15 minutes. Find how long will it take for the metal ball to reach a temperature of 40°C.

Sol:-

$t_1 = 100$

$t_2 = 30^\circ$

$T = 70^\circ$

$$T = t_2 + (t_1 - t_2)\, e^{-kt}$$

$$70 = 30 + (100 - 30)\, e^{-k}$$

$$70 = 30 + (70)\, e^{-15k}$$

$$70 e^{-15k} = 70 - 30$$

$$70 e^{-15k} = 40$$

$$e^{-15k} = \cancel{\frac{40}{70} = 0.571} \qquad \frac{70}{40} = 1.75$$

$$e^{-15k} = 1.75$$

$$-15k = \log(7/4) = \log(1.75)$$

$$15k = 0.5596$$

$$k = \frac{0.5596}{15} = 0.0373$$

$$(40) = 30 + 70 e^{-0.0373}$$

$$70 e^{-0.0373} = 40 - 30$$

$$70 e^{-0.0373} = 10$$

$$e^{-0.0373} = 7$$

$$0.0373t = 1.945$$

$$t = \frac{1.945}{0.0373} = 52.144$$

28/12/18

L-R Circuits

A Series circuit with resistance 'R' Inductance 'L' and electromotive force and force E is governed by the differential equation.

$$L\frac{di}{dt} + Ri = E$$

where L & R are constants and initially the current 'i' is zero.
Find the current at any time 't'

Sol:

$$L\frac{di}{dt} + Ri = E$$

$\div L$

$$\frac{di}{dt} + \frac{R}{L}i = \frac{E}{L}$$

$$P = \frac{R}{L}, \quad Q = \frac{E}{L}$$

$$e^{\int P dt} = e^{\int R/L\, dt} = e^{Rt/L}$$

$$i.\, e^{\int P dt} = \int Q e^{\int P dt}\, dt + C$$

$$i.e^{RT/L} = \int \frac{R}{L} e^{Rt/L}\, dt + c$$

$$i.e^{Rt/L} = \frac{E}{L}\left[\frac{e^{Rt/L}}{R/L}\right] + c$$

$$ie^{Rt/L} = \frac{E}{\cancel{L}} \times \frac{\cancel{L}}{R} e^{Rt/L} + c$$

$\div$ throughout by $e^{Rt/L}$

$$i = \frac{E}{R} + \frac{C}{e^{Rt/L}}$$

$$i = \frac{E}{R} + Ce^{-Rt/L}$$

$i = 0,\ t = 0$

$$0 = \frac{E}{R} + Ce^{0}$$

$$C = -\frac{E}{R}$$

$$i = \frac{E}{R} - \frac{E}{R}e^{-Rt/L}$$

$$i = \frac{E}{R}\left[1 - e^{-Rt/L}\right]$$

(#) An inductance 2 Henry and the resistance 20 Ω are connected in series with emf E (volts). If the current is initially zero when t = 0. Find the current at the end of 0.01 sec if E = 100 V.

Sol:-

$$i = \frac{E}{R}\left[1 - e^{-Rt/L}\right]$$

$$= \frac{\cancel{100}^{5}}{\cancel{20}}\left[1 - e^{-\cancel{20}^{10}\times 0.01/\cancel{2}}\right]$$

$i_{(t=0.01)} = 5\left[\cancel{0.1}\right]\left[1-0.1\right]$ $\qquad 1/e^{0.1} = 0.406$ $\cancel{0.09}$

$5(0.9)$

$= 0.47$

$$i_{(t=0.01)} = 5\left[1 - 0.1\right]$$

$$= 5\left[1 - 0.9048\right]$$

$$i = 5\left[0.0952\right]$$

$$i = 0.47A$$

29/12/18

Equation solvable for 'P'

(1) $y\left(\frac{dy}{dx}\right)^2 + (x-y)\frac{dy}{dx} - x = 0$

$$yP^2 + (x-y)P - x = 0$$

$$a = y,\ b = (x-y),\ c = -x$$

$$P = \frac{-(x-y) \pm \sqrt{(x-y)^2 + 4xy}}{2y}$$

$$= \frac{(y-x) \pm \sqrt{x^2 + y^2 - 2xy + 4xy}}{2y}$$

$$= \frac{(y-x) \pm \sqrt{(x+y)^2}}{2y}$$

$$P = \frac{(y-x) \pm (x+y)}{2y}$$

$$P = \frac{y - \cancel{x} + \cancel{x} + y}{2y}$$

$$\boxed{P = 1}$$

$$P = \frac{\cancel{y} - x - x + \cancel{-y}}{2y}$$

$$\boxed{P = -\frac{x}{y}}$$

SURYA Gold

Date ____ Page ____

1) $\frac{dy}{dx} = 1$

$$dy = dx$$
$$\int dy = \int dx + c$$
$$y = x + c$$
$$\boxed{y - x = c} \text{ (or) } \boxed{y - x - c = 0}$$

2) $\frac{dy}{dx} = \frac{-x}{y}$

$$y\,dy = -x\,dx$$
$$\int y\,dx + \int x\,dx = c$$
$$\frac{y^2}{2} + \frac{x^2}{2} = c \Rightarrow x^2 + y^2 = 2c$$

$$(x^2 + y^2 - 2c) = 0$$
$$(y - x - c)(x^2 + y^2 - 2c) = 0$$

(2) Solve for

$$xy\left(\frac{dy}{dx}\right)^2 - (x^2+y^2)\frac{dy}{dx} + xy = 0$$

$$xyp^2 - (x^2+y^2)p - xy = 0$$
$$a = xy \quad b = -(x^2+y^2), \quad c = -xy$$

$$P = \frac{(x^2+y^2) \pm \sqrt{[(x^2+y)^2]^2 \mp 4x^2y^2}}{2xy}$$

$$= \frac{(x^2+y^2) \pm \sqrt{x^4+y^4+2x^2y^2 - 4x^2y^2}}{2xy}$$

$$= \frac{(x^2+y^2) \pm \sqrt{(x^2-y^2)^2}}{2xy}$$

SURYA Gold

Date______ Page______

$$P = \frac{(x^2+y^2) \pm (x^2-y^2)}{2xy}$$

$$P = \frac{x^2+\cancel{y^2}+x^2-\cancel{y^2}}{2xy}$$

$$\boxed{P = \frac{x}{y}}$$

$$P = \frac{\cancel{x^2}+y^2-\cancel{x^2}+y^2}{2xy}$$

$$\boxed{P = \frac{y}{x}}$$

1) $\frac{dy}{dx} = \frac{x}{y}$

$ydx = xdx$

$\int ydy - \int xdx = c$

$$\frac{y^2}{2} - \frac{x^2}{2} = c$$

$$\boxed{y^2 - x^2 - 2c = 0}$$

2) $\frac{dy}{dx} = \frac{y}{x}$

$\frac{dy}{y} = \frac{dx}{x}$

$\int \frac{dy}{y} - \int \frac{dx}{x} = K$

$\log y - \log x = K$

$\log (y/x) = \log c$

SURYA Gold

Date________ Page________

$y/x = c \Rightarrow \boxed{y - cx = 0}$

$$\boxed{(y^2 - x^2 - 2c)(y - cx) = 0}$$

$\cancel{p^2 - 2p\sinh x - 1 = 0}$

(8) $p^2 - 2p\sinh x - 1 = 0$

$a = 1, \quad b = -2p\sinh x, \quad c = -1$

$$= \frac{2p\sinh x \pm \sqrt{(-2p\sinh x)^2 - 4 \times p^2 \times (-1)}}{2 \times 1}$$

$$p = \frac{2\sinh x \pm \sqrt{2^2 \sin^2 hx + 4}}{2}$$

$$p = \frac{2\sinh x \pm \sqrt{\cos^2 hx}}{2}$$

$$p = \frac{4\sinh x + 2}{2} \quad \frac{2\sinh x \pm 2\cosh x}{2}$$

$$p = \frac{\cancel{2}(\sinh x + 1)}{\cancel{2}}$$

$p = \sinh x + \cosh x$

$p = \sinh x - \cosh x$

1) $\frac{dy}{dx} = \sinh x + \cosh x$

$\int dy = \int \sinh x + \cosh x \, dx$

$y = \cosh x + \sinh x + c$

$y - \cosh x - \sinh x - c = 0$

SURYA Gold

Date ______ Page ______

$$\frac{dy}{dx} = \sinh x - \cos x$$

$$\int dy = \int \sinh x - \cosh x \, dx$$

$$y = \cosh x - \sinh x + c$$

$$y - \cosh x + \sinh x - c = 0$$

$$\cos(y - \cosh x - \sinh x - c)\,(y - \cosh x + \sinh x - c) = 0$$

9/1/19

S.T the equation $y = Px + f(P)$ → is the D.E is ~~for~~ Clairaut's equation & hence obtain the general and singular solution.

(1) $xp^2 + Px - Py + 1 - y = 0$

$$y = xp^2 + Px - Py + 1 - y = 0 \quad \text{~~is the D.E~~}$$

$$= xp(P+1) - Py + 1 = 0$$

$$= xp - y + \frac{1}{P+1} = 0$$

$$y = xp + \frac{1}{P+1}$$

$$y = Px + f(P)$$

$$y = Cx + \frac{1}{C+1} \longrightarrow (1)$$

diff w.r.t 'C'

$$0 = x - \frac{1}{(C+1)^2}$$

$$0 = \frac{x(C+1)^2 - 1}{(C+1)^2}$$

$$x(C+1)^2 = 1$$

$$C+1 = 1/\sqrt{x}$$

$$C = \frac{1}{\ldots} - 1$$

SURYA Gold

Date______ Page______

$$y = \left(\frac{1}{\sqrt{x}} - 1\right)x + \frac{1}{\frac{1}{\sqrt{x}} \cancel{-1} \cancel{+1}}$$

$$= \sqrt{x} - x + \sqrt{x}$$

$$y = 2\sqrt{x} - x$$

$$x + y = 2\sqrt{x}$$

$$(x+y)^2 = 4x$$

(2) $xp^2 - Py + KP + a = 0$

$$\frac{xp^2 + KP + a}{P} = y$$

$$\frac{P(xp + k) + a}{P}$$

$\cancel{P(xP+}$ $y = KP + \frac{a}{P} + K$

$$y = Px + f(P)$$

$$y = Cx + \frac{1}{C+1}$$

$\cancel{0 = Cx - \frac{a}{C^2}}$ diff w.r.t 'c'

$$0 = x - \frac{a}{c^2}$$

$$0 = \frac{xc^2 - a}{c^2}$$

$$xc^2 - a = 0$$

$$xc^2 = a$$

$$c^2 = \frac{a}{x}$$

$$c = \sqrt{\frac{a}{x}}$$

SURYA Gold

Date______ Page______

$$y = \sqrt{\frac{a}{x}} + \frac{a}{\sqrt{a}} + K$$

$$y = \sqrt{x} \cdot \sqrt{a} + \sqrt{a} \cdot \sqrt{x} + K$$

$$y = K + 2\sqrt{a}\sqrt{x} \qquad y - K = 2\sqrt{ax}$$

$$(y-K)^2 = 4a\cancel{x}$$

(3) Solve $(y - Px)^2 = 4P^2 + 9$

(4) Solve the equ^n $(Px - y)(Py + x) = 2P$ by reducing into clairaut's form taking the substitution $X = x^2$, $Y = y^2$

$$\frac{dX}{dx} = 2x, \quad \frac{dY}{dy} = 2y$$

$$y = \frac{dy}{dx} = \frac{dy}{dY} \cdot \frac{dY}{dX} \cdot \frac{dX}{dx}$$

$$\frac{dY}{dX} = P$$

$$= \frac{1}{\cancel{2}y} \cdot P \cdot \cancel{2}x$$

$$P = \frac{x}{y} P$$

$$(Px - y)(Py + x) = 2P$$

$$(P\sqrt{X} - \sqrt{Y})(P\sqrt{Y} + \sqrt{X}) = 2P$$

$$\left(\frac{x}{y} P\sqrt{X} - \sqrt{Y}\right)\left(\frac{x}{y} P\sqrt{Y} + \sqrt{X}\right) = 2\frac{x}{y} P$$

SURYA Gold

Date ______ Page ______

$$\left(\frac{\sqrt{X}}{\sqrt{Y}} P\sqrt{X} - \sqrt{Y}\right)\left(\frac{\sqrt{X}}{\cancel{\sqrt{Y}}} P\cancel{\sqrt{Y}} + \sqrt{X}\right) = \frac{2\sqrt{X}\,P}{\sqrt{Y}}$$

$$\left(\frac{PX - Y}{\sqrt{Y}}\right)\left(P\sqrt{X} + \sqrt{X}\right) = \frac{2\sqrt{X}\,P}{\sqrt{Y}}$$

$$\left(\frac{PX - Y}{\cancel{\sqrt{Y}}}\right)(P+1)\sqrt{X} = \frac{2\sqrt{X}\,P}{\cancel{\sqrt{Y}}}$$

$$(PX - Y)(P+1)\cancel{\sqrt{X}} = 2\cancel{\sqrt{X}}\,P$$

$$(PX - Y)(P+1) = 2P$$

$$PX - Y = \frac{2P}{P+1}$$

$$Y = PX - \frac{2P}{P+1}$$

$$Y = cx - \frac{2c}{c+1}$$

3/1/19

(5) Solve $e^{4x}(P-1) + e^{2y}.P^2 = 0$ by using the Substitution $u = e^{2x}$ and $v = e^{2y}$

$$\frac{du}{dx} = 2e^{2x}$$

$$e^{4x}(P-1) + e^{2y}.P^2 = 0$$

$$= e\ (P-1) + e^{v}(P^2) = 0$$

$$\frac{dv}{dy} = 2e^{2y}$$

$$P = \frac{dy}{dx} = \frac{dy}{dv}\ \frac{dv}{du}\ \frac{du}{dx} \quad \text{let } P = \frac{dv}{du}$$

$$P = \frac{1}{2e^{2y}}\ P\,2e^{2x}$$

$$\left[\frac{x^5}{5}\right]_1^2$$

$$\frac{2^5}{5} - \frac{1}{5}$$

$$\frac{32}{5} - \frac{1}{5}$$

$$\frac{31}{5}$$

$$P = \frac{u}{v} p$$

$$\Rightarrow e^{4x}(p-1) + e^{2y} p^2 = 0$$

i.e., $u^2\left(\frac{v}{u} p - 1\right) + v \cdot \frac{u^2}{v^2} p^2 = 0$

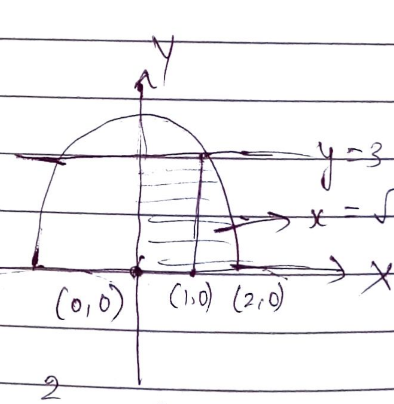
$$\frac{u^2(up - v)}{v} + \frac{u^2 p^2}{v} = 0$$

$$up - v + p^2 = 0$$

$v = pu + p^2$ is in the Clairaut's equⁿ

Change the order of integration and evaluate

$$\int_0^3 \int_0^{\sqrt{4-y}} (x+y)\, dy\, dx$$



$y = 0$ to 3

$x = 0$ to $\sqrt{4-y}$

$$I = \int_{x=0}^{1} \int_{y=0}^{3} (x+y)\, dy\, dx + \int_{x=1}^{2} \int_{y=0}^{4-x^2}$$

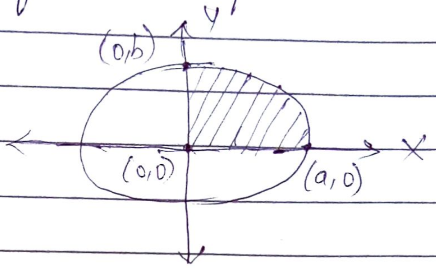
5/1/19

SURYA Gold

Date ______ Page ______

Find the centre of gravity of a lamina in the shape of a quadrant of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



$$\frac{y^2}{b^2} = 1 - \frac{x^2}{a^2}$$

$$y^2 = \frac{b^2}{a^2}(a^2 - x^2)$$

$$y = \frac{b}{a}\sqrt{a^2 - x^2}$$

$$\int_0^a \int^{b/a\sqrt{a^2-x^2}}$$

$$m, n = \iint_A \rho \, dx \, dy$$

$$= \int_0^a \int_0^{b/a\sqrt{a^2-x^2}} xy \, dy \, dx$$

$$= \int_0^a x \left[\frac{y^2}{2}\right]_0^{b/a\sqrt{a^2-x^2}}$$

$$= \frac{1}{2}\int_0^a x \left(\frac{b}{a}\sqrt{a^2-x^2}\right)^2$$

$$= \frac{1}{2}\int_0^a x \left(\frac{b^2}{a^2}(a^2-x^2)\right)$$

$$= \frac{b^2}{a^2}\int_0 x(a^2-x^2)\,dx$$

$$= \frac{b^2}{2a^2}\left[a^2\left(\frac{x^2}{2}-\frac{x^4}{4}\right)\Big|_0^a\right]$$

$$= \frac{b^2}{2a^2}\left[a^2\left(\frac{a^2}{2}-\frac{a^4}{4}\right)\right]$$

$$= \frac{b^2}{2\cancel{a^2}}\cancel{a^2}\left[\frac{1}{2}-\frac{1}{4}\right]$$

$$= \frac{b^2a^2}{2\times 4}$$

$$= \frac{b^2a^2}{8}$$

$$\bar{x} = \iint_A x\rho\, dy\, dx$$

$$= \int_0^a\int_0^{b/a\sqrt{a^2-x^2}} x^2 y\, dy\, dx$$

$$= \int_0^a\int_0 x^2\left[\frac{y^2}{2}\right]_0^{b/a\sqrt{a^2-x^2}} dx$$

$$= \frac{1}{2}\int_0^a x^2\left(b^2/a^2\sqrt{a^2-x^2}\right)$$

$$= \frac{b^2}{2a^2}\int_0^a x^2a^2 - x^4\, dx$$

$$= \frac{b^2}{2a^2}\left[a^2\frac{x^3}{3}-\frac{x^5}{5}\right]_0^a = \left[\frac{a^5}{3}-\frac{a^5}{4}\right]\frac{b^2}{2a^2}$$

$$= \frac{a^5b^2}{2a^2}\left[\frac{1}{3}-\frac{1}{5}\right]$$

SURYA Gold

Date ____ Page ____

$$\frac{b^2 a^{\cancel{5}\,3}}{2\cancel{a^2}}\left[\frac{1}{3}-\frac{1}{5}\right]$$

$$= \frac{a^3 b^2}{15}$$

$$\bar{x} = \frac{a^3 b^2}{15} \Big/ mA$$

$$\bar{x} = \frac{a^3 b^2/15}{a^2 b^2/8} = \frac{a^{\cancel{3}}\cancel{b^2}}{15} \times \frac{8}{\cancel{a^2}\cancel{b^2}}$$

$$= \frac{8a}{15}$$

Illy

$$\boxed{y = \frac{8b}{15}}$$

(#)
$$\int_{x=0}^{1}\int_{y=0}^{\sqrt{1-x^2}}\int_{z=0}^{\sqrt{1-x^2-y^2}} \frac{dz\,dy\,dx}{\sqrt{1-x^2-y^2-z^2}}$$

$$= \int_0^1\int_0^{\sqrt{1-x^2}}\int_0^{\sqrt{1-x^2-y^2}} \frac{dz\,dy\,dx}{\left(\sqrt{1-x^2-y^2}\right)^2 - z^2}$$

$$= \int_0^1\int_0^{\sqrt{1-x^2}}\int_0^{\sqrt{1-x^2-y^2}} dz\,dy\,dx$$

$$= \int_0^1\int_0^{\sqrt{1-x^2}}\int_0^{\sqrt{1-x^2-y^2}\ k} \frac{dz\,dy\,dx}{k^2 - z^2}$$

SURYA Gold

Date ____ Page ____

$$= \int_0^1 \int_0^{\sqrt{1-x^2}} \left[\sin^{-1} \frac{z}{k} \right]_0^k dy\, dx$$

$$= \int_0^1 \int_0^{\sqrt{1-x^2}} \left(\frac{\pi}{2} - 0 \right) dy\, dx$$

$$= \frac{\pi}{2} \int_0^1 \int_0^{\sqrt{1-x^2}} dy\, dx$$

$$= \frac{\pi}{2} \int_0^1 \left[y \right]_0^{\sqrt{1-x^2}} dx$$

$$= \frac{\pi}{2} \int_0^1 \sqrt{1-x^2}\, dx$$

$$= \frac{\pi}{2} \left[\frac{x\sqrt{a^2-x^2}}{2} + \frac{a^2}{2} \sin^{-1}\left(\frac{x}{a}\right) \right]_0^1$$

$$= \frac{\pi}{2} \left[0 + \frac{1}{2} \frac{\pi}{2} \right.$$

$$= \frac{\pi}{2} \left[\frac{\pi}{4} \right]$$

$$\boxed{I = \frac{\pi^2}{8}}$$